VOLUME 84 NO. SU1
APRIL 1958

JOURNAL of the

Surveying and Mapping Division

PROCEEDINGS OF THE

AMERICAN SOCIETY OF CIVIL ENGINEERS

BASIC REQUIREMENTS FOR MANUSCRIPTS

This Journal represents an effort by the Society to deliver information to the reader with the greatest possible speed. To this end the material herein has none of the usual editing required in more formal publications.

Original papers and discussions of current papers should be submitted to the Manager of Technical Publications, ASCE. The final date on which a discussion should reach the Society is given as a footnote with each paper. Those who are planning to submit material will expedite the review and publication procedures by complying with the following basic requirements:

1. Titles should have a length not exceeding 50 characters and spaces.

2. A 50-word summary should accompany the paper.

3. The manuscript (a ribbon copy and two copies) should be double-spaced on one side of 8½-in. by 11-in. paper. Papers that were originally prepared for oral presentation must be rewritten into the third person before being submitted.

4. The author's full name, Society membership grade, and footnote reference stating present employment should appear on the first page of the paper.

5. Mathematics are reproduced directly from the copy that is submitted. Because of this, it is necessary that capital letters be drawn, in black ink, ⅛-in. high (with all other symbols and characters in the proportions dictated by standard drafting practice) and that no line of mathematics be longer than 6½-in. Ribbon copies of typed equations may be used but they will be proportionately smaller in the printed version.

6. Tables should be typed (ribbon copies) on one side of 8½-in. by 11-in. paper within a 6½-in. by 10½-in. invisible frame. Small tables should be grouped within this frame. Specific reference and explanation should be made in the text for each table.

7. Illustrations should be drawn in black ink on one side of 8½-in. by 11-in. paper within an invisible frame that measures 6½-in. by 10½-in.; the caption should also be included within the frame. Because illustrations will be reduced to 69% of the original size, the capital letters should be ⅛-in. high. Photographs should be submitted as glossy prints in a size that is less than 6½-in. by 10½-in. Explanations and descriptions should be made within the text for each illustration.

8. Papers should average about 12,000 words in length and should be no longer than 18,000 words. As an approximation, each full page of typed text, table, or illustration is the equivalent of 300 words.

Further information concerning the preparation of technical papers is contained in the "Technical Publications Handbook" which can be obtained from the Society.

Reprints from this Journal may be made on condition that the full title of the paper, name of author, page reference (or paper number), and date of publication by the Society are given. The Society is not responsible for any statement made or opinion expressed in its publications.

This Journal is published by the American Society of Civil Engineers. Publication office is at 2500 South State Street, Ann Arbor, Michigan. Editorial and General Offices are at 33 West 39 Street, New York 18, New York. $4.00 of a member's dues are applied as a subscription to this Journal.

AT,BD,CP,CO,HW,IR,SU,WW,PI.

Journal of the
SURVEYING AND MAPPING DIVISION
Proceedings of the American Society of Civil Engineers

SURVEYING AND MAPPING DIVISION
EXECUTIVE COMMITTEE
Carl M. Berry, Chairman; Milton O. Schmidt, Vice-Chairman;
Oscar C. J. Marshall; Earle J. Fennell; Franklin R. Gossett, Secretary

COMMITTEE ON PUBLICATIONS
Oscar C. J. Marshall, Chairman

CONTENTS

April, 1958

Papers

Number

Highway and Bridge Surveys: Reconnaissance
Progress Report of the Committee on Highway
and Bridge Surveys of the Surveying and Mapping Division. 1593

Geodetic Control for Tropospheric Scatter Antennas
by Max O. Laird and Ing. Antonio Aguilar 1594

Short Methods in Adjustment of Observations
by M. V. Smirnoff and Paul E. Wylie 1595

Discussion 1605

The Geodimeter and Tellurometer
by Austin C. Poling. 1617

Copyright 1958 by the American Society of Civil Engineers.

Paper 1593 April, 1958 SU 1

Journal of the
SURVEYING AND MAPPING DIVISION
Proceedings of the American Society of Civil Engineers

HIGHWAY AND BRIDGE SURVEYS: RECONNAISSANCE[a]

Progress Report of the Committee on Highway and Bridge Surveys of the Surveying and Mapping Division
(Proc. Paper 1593)

INTRODUCTION

Purpose and Scope of Reconnaissance

Reconnaissance is the process of evaluating the feasibility of one or more possible routes for a highway between specific points. Often many miles separate these points. Service requirements of the termini and intermediate points are determined in advance by traffic studies. Consistent with proper service, selection of an approximate route involves consideration of roadway design standards, topography, geology and land-use.

The results of reconnaissance are embodied in reports and maps, which guide those who execute the preliminary survey in their more detailed investigations.

Importance of Reconnaissance

All-new locations being the rule rather than the exception in today's Federal Interstate Highway program, reconnaissance assumes great importance in the free-road field.

Good reconnaissance can be the largest single money-saving phase in the construction of a new road. With construction costing as much as a half-million dollars a mile, the location that is only ten percent cheaper than the more obvious route will save many times the expense of modern reconnaissance. Hence, the engineer should make ample provision for this first-stage investigation.

Fortunately today there are many sources of information and new techniques that give the highway engineer a pronounced advantage over reconnaissance engineers of the past. Extensive availability of topographic maps and broader existing aerial photographic coverage give the locating engineer

Note: Discussion open until September 1, 1958. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. Paper 1593 is part of the copyrighted Journal of the Surveying and Mapping Division, Proceedings of the American Society of Civil Engineers, Vol. 84, No. SU 1, April, 1958.

a. This paper will form the basis for a chapter in a proposed ASCE Manual of Engineering Practice.

much assistance. Furthermore the science of photogrammetry stands ready to provide special data of many kinds.

Sources of Information

Existing maps and aerial photographs normally are the first tools of the reconnaissance engineer. The status of coverage can be determined by inquiry addressed to the

Map Information Office
U. S. Geological Survey
Washington 25, D. C.

In addition to listing U.S.G.S. maps, aerial photographic material, and geodetic control data, the Map Information Office maintains records of maps and aerial photographic coverage of other Federal agencies, including the geodetic control available from the U. S. Coast and Geodetic Survey. Information on aerial mosaics, soil and subsurface maps, and unclassified strategic maps also can be obtained through that office.

The status of published topographic maps is shown on U.S.G.S. index maps available for each state. These indexes show quadrangle sizes, names, scales, and dates of survey. Many of the newer quadrangle maps are published at 1:24,000 scale (1 in. = 2000 ft.) with a 5, 10, or 20-ft. contour interval. About 40 percent of the United States is covered with topographic quadrangle maps published at scales of 1:62,500 (1 in. = approximately 1 mile) or larger, and which are considered adequate for general purposes. A status map for the entire country depicts the national mapping effort, both completed and in progress.

Photographic coverage is shown on another national index map available from the Map Information Office. Scales and dates of photography are not shown, but the name of the agency holding the film is given.

"In-progress" mapping information frequently is available from the producing agencies, and the partially-completed work often can be obtained.

Other sources of useful material are railroads (profiles, land corner ties), state geological offices (maps, reports), and state universities. A few states, such as Maryland and Washington, have their own map information offices.

Search also should be made for horizontal and vertical control data applicable to the study area. In addition to the U. S. Coast and Geodetic Survey and the U. S. Geological Survey geodetic control data, which are readily available from those agencies, a search should be made for possible control-network extensions by other Federal agencies or from State and local agencies. Special effort should be made to utilize state plane coordinates for horizontal control and the 1929 mean sea level datum for vertical control.

Conduct of Reconnaissance

Study of Available Data

Reconnaissance begins with a "paper study" of an area four-tenths to six-tenths as wide as the distance between the termini. Ridges, water courses, and land-use are examined first. Low points, or passes in the ridges, become potential fixed points in the location, as do stream-crossing sites that

afford suitable topography for approaches. Thickly settled areas, as well as swamps and other ground unsuited to economical construction, should be avoided. Sites for necessary grade-separation structures should be found along intersecting roads and railroads, and utility-relocation problems should be anticipated.

Discovery of the above controlling factors, together with rapid appraisal of intervening grades and horizontal alignments, is possible with the modern topographic map. Photographs supplement the map, to a greater or lesser extent depending on its quality. Stereoscopic techniques can afford quantitative data and, when applied by a skilled photo-interpreter, they can yield significant soils and subsurface information.

Field Inspection

Next, cognizant of the questions generated by the map study, the reconnaissance engineer goes to the field. The best way for him to get a first look at key points and intervening alignments is to fly over them. Much of the engineer's aerial-visual data will be sufficient to accomplish his appraisal of the terrain. He then can restrict his ground study to considerations of unusual topographic features, hydrology, the nature of some man-made works, and subsurface conditions. If the only available maps do not portray topography, he should obtain approximate elevations of key points with a modern surveying altimeter.

The engineer should utilize all available survey control as he executes the reconnaissance. Bench marks are reference points for altimeter observation. Triangulation and traverse points may be tied approximately to points that will be distinguishable in aerial photographs.

Trial Locations

Returning to his office, the engineer makes one or more paper locations of feasible routes. If he is satisfied with the quality and extent of his data, he may make the necessary economic comparison and proceed with the preparation of the reconnaissance report.

New Aerial Photographs

Frequently, it is desirable to procure new photography along each route. These photographs show current land-use. Moreover, the new photos are likely to be at a larger scale than previous photos which were not made specifically for a like project. Hence, they will give better detail for qualitative and quantitative analyses.

Photograph Mosaics

Mosaics made from aerial photographs are useful throughout the highway engineering study. Beyond the reconnaissance phase, they are visual aids in conferences of highway officials and at public hearings. Design engineers seeking a better understanding of field conditions regularly refer to the mosaics.

Map-Mosaic Analysis

In reconnaissance, the mosaic is of particular value when it is viewed in

conjunction with a topographic map. A frequently-used highway mosaic scale is 1 in. = 1000 ft. Made to cover a 3-mile-wide band (about 16 in.), when used for reconnaissance purposes, this mosaic is overlaid by a topographic map that has been enlarged to the same scale and printed on transparent material. Depending on the location of possible lines, one or more mosaic bands, each covering all or part of the route length, may be assembled and overlaid. Trial lines are sketched on the mosaics and profiles are plotted directly from the contours of the overlay.

When grade studies on these profiles indicate alignments that satisfy the highway's design standards, such alignments are compared in terms of cost and road user benefits.

The economical location resulting from the above analysis should be shown on the mosaic as a strip within which preliminary surveyors will gather data for final-alignment studies. Where aerial mapping is anticipated, a liberal width of strip should be considered, say, one-half mile.

Other mosaic scales and route-band widths may apply in specific cases. Suitable scales and other photographic applications that are useful to the reconnaissance engineer are discussed comprehensively and ably in Highway Research Board Bulletin No. 157, "Photogrammetry and Aerial Surveys - A Symposium" 1957.

Aerial Photography and Photogrammetry

The least expensive route-band mosaic is obtained from a single line of flight by the photographic aircraft. Limitations on width of coverage are imposed by the aircraft's service ceiling, focal length of the camera and the amount of photo-enlargement available without loss of definition. Edge zones of photos are usually omitted from mosaics because of unavoidable and permissible position errors of the aircraft, and because of possible edge distortion of image.

In the foregoing example of a 1 in. = 1000 ft. mosaic, the flight height with a 6-in. focal length camera could be 15,000 ft., which is a practicable altitude for most photographic aircraft. The scale of the resulting 9 x 9-in. photographs would be 1:30,000, or 1 in. = 2,500 ft. Two-and-one-half times enlargement to 1 in. = 1000 ft. is easily possible without significant loss of definition (5x is an upper limit), giving a total picture width of 4.3 miles and a practicable mosaic width of 3 miles.

Mosaic is not a Map

The single-flight-line mosaic is satisfactory for all mosaic uses that have been cited. However, the mosaic makes no pretenses to possessing map precision. Distances can be scaled only approximately, and there will be minor mismatches of individual photographs.

Multiple Flight Lines

In some reconnaissance situations individual photographs of single-flight-line mosaics will have limitations that may make a narrower band or multiple flight lines preferable. Either recourse will yield a larger photo contact scale. Relatively small scale curtails the possible extent of photo analysis and interpretation. Where terrain is thickly settled, larger scales are required for adequate examination.

Reconnaissance Mapping

Where published maps are inferior or non-existent, it may be desirable to prepare for medium-scale mapping by taking mosaic photos that also can be used for stereoplotting. The largest map scale that is likely to be needed at this stage is 1 in. = 200 ft., which normally would be plotted from a contact scale of 1 in. = 1000 ft.

Such dual-purpose photography, while it may suffer from lack of placement of the flight line in its most efficient position for mapping, may have the advantage of saving a photographic season. To be usable for mapping, photos must show the ground. Foliage obscures it and snow camouflages it, so in most areas there are definite seasons for mapping flights.

Ground control for reconnaissance photogrammetry may be confined to as few as one horizontal point about every fifth photograph and two vertical points about every third photograph. Intervening control can be "bridged in" with stereoplotting equipment.

Stereoscopic Equipment

Short of mapping, which is performed by a commercial air mapping firm or by a highway agency's aerial survey department, the reconnaissance engineer has at his disposal stereoscopic devices that can yield rough quantitative data from pictures.

A parallax bar, used in conjunction with a small stereoscope and with only the approximate control of the nominal photo scale, can measure elevation differences within limited areas of a pair of photos to about (1/100)S to (1/50)S where S is the scale number when the scale is stated in the form 1 in. = S ft. A magnifying mirror stereoscope is recommended for this work, although a pocket lens stereoscope will do.

Air-Photo Interpretation

Qualitative data, too, can be obtained with the stereoscope. With this instrument, suitable photos and favorable geological conditions, the skilled photo-interpreter can:

a) Locate sand and gravel deposits, borrow pits and rock areas suitable for quarrying;

b) Rate soil bearing capacities as good, fair and poor;

c) Outline soils as to their texture, and estimate the depths of organic deposits;

d) Classify bedrock as to physical types and their relative depth below surface;

e) Estimate depths of glacial drift and windblown material;

f) Estimate hydrological factors;

g) Delineate active and potential areas involving problems such as sinkholes, slides, rock falls, and frost-heave;

h) Locate underground utilities placed by cut-and-cover methods.

Spot field checks of photo-interpretation results are desirable.

Reconnaissance Report

With the wealth of data resulting from the application of the foregoing investigative techniques, the reconnaissance engineer can estimate approximate costs and determine literally the best available route.

Results of his studies are presented in a reconnaissance report. In its barest essentials this report states the criteria (service and geometrics) to be satisfied by the project, describes its chosen line, and presents a tentative estimate of its cost. Maps and mosaics are important pictorial adjuncts.

Report is a Public Relations Device

Major highway projects, however, deserve a comprehensive report. In the case of Interstate Highways, where a public hearing is required for each project, the report should be designed as a public-relations device—a document intended to convince highway officials, legislators, and the general public that the best location has been found. The report should cover such matters as the exhaustive nature of the study, concern for landowners' interests, highway safety, and project benefits to both the road user and the communities served.

Maps and Mosaics

Accompanying material should include a key map, mosaic sheets showing the location, profiles, typical roadway sections and typical structures. Aerial oblique photographs also can be useful to show where the road will be located at critical places.

Control Data

For the benefit of those conducting subsequent surveys, an appendix describing all existing control accessible to the route will be helpful and time-saving.

Report Makeup

The comprehensive report should begin with a Summary, which in essence "writes the newspaper story" about the proposed project. When presented with a key map, the Summary should be comprehensive in giving project-benefit information and be brief in its technical description.

If there are unavoidable difficult situations affecting the public—a neighborhood displacement, for instance—they should be cited in a forthright manner.

All points of the summary should be amply supported in the report's main body. The following is a typical reconnaissance report outline:

I SUMMARY (Conclusions and Recommendations)

II REPORT

- A Introduction
  1. Purpose of Reconnaissance Survey
  2. Traffic Survey Results
  3. Study Methods
  4. Design Criteria
- B General Alignment Details
  1. Choice of Route

2. Service to Communities
3. Interchange Locations
4. Land Use and Severance
5. Safety Considerations
6. Drainage
7. Soils and Geology

C Project Cost Estimate
1. Roadway
2. Structures
3. Right-of-way
4. Utility Relocation

Respectfully submitted

Oliver R. Bosso — A. A. Katterhenry
R. H. Dodds — John G. McEntyre
R. A. Haber — C. A. Rothrock
Peter A. Hakman — L. R. Schureman
Milton O. Schmidt, Chairman

Committee on Highway and Bridge
Surveys of the Surveying and
Mapping Division

Paper 1594 April, 1958 SU 1

Journal of the
SURVEYING AND MAPPING DIVISION
Proceedings of the American Society of Civil Engineers

GEODETIC CONTROL FOR TROPOSPHERIC SCATTER ANTENNAS[a]

Max O. Laird,[1] M. ASCE and Ing. Antonio Aguilar,[2] A.M. ASCE
(Proc. Paper 1594)

SYNOPSIS

A wide-band high-power forward tropospheric scatter radio system, employing both space and frequency diversity and 60-foot parabolic reflector antennas, was constructed to provide additional public overseas telephone service and a television network link between the United States and Cuba. Construction was completed during the past year, the new facilities being placed in service September 12, 1957. One radio station, located in the Everglades about 10 miles southeast of Florida City, is owned and operated by the American Telephone and Telegraph Company. The corresponding station is on a hill overlooking Guanabo, Cuba and is owned and operated by the Radio Corporation of Cuba. The two narrow-beam parabolic reflector antennas at each end are aimed at the effective radio horizon and in the azimuthal direction to the corresponding antennas at the far end of the 300 km beyond-the-horizon path. Aiming was effected by computing azimuth, back azimuth and distance between geographic positions for the antennas determined by triangulation expansions from the existing geodetic control networks in the United States and in Cuba. Second-order accuracy was used for the Florida expansion from completely adjusted first-order triangulation of the U. S. Coast and Geodetic Survey. Third-order accuracy was used for the Cuba expansion from first-order triangulation established jointly by the Cuban Institute of Cartographic and Cadastral Surveys and the Interamerican Geodetic Survey, for which only a preliminary adjustment and a solution within about 0.01 second of latitude and longitude are currently available. The geodetic surveys, in addition to controlling the aiming of the antennas serve also to furnish State Plane Coordinates and to control all boundary, topographic and construction surveys

Note: Discussion open until September 1, 1958. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. Paper 1594 is part of the copyrighted Journal of the Surveying and Mapping Division Proceedings of the American Society of Civil Engineers, Vol. 84, No. SU 1, April, 1958.

a. Presented at meeting of the ASCE, New York, N. Y., October, 1957.

1. Western Electric Co., Defense Projects Div., New York, N. Y.
2. Director Topographic Div., Saenz-Cancio-Martin, Engrs.; and Alvarez and Gutierrez, Archts.; Havana, Cuba; consultants to Radio Corporation of Cuba.

throughout the anticipated long life of the radio stations. The procedure and results of the geodetic control surveys are described. Reference is made to other technical papers covering the radio system engineering and equipment design.

General Description and Requirements for Geodetic Control

Public overseas telephone service between the United States and Cuba is currently furnished by 4 single and 1 twin coaxial submarine cable systems between Key West and Havana. The rate of growth of telephone traffic is so great as to make it uneconomical to meet future growth requirements by adding additional cables, each capable of furnishing only something like 36 one-way circuits at the current stage of cable development. A wide-band system capable of handling hundreds of telephone conversations and suitable also for television network service is required. Recent developments by the Bell Telephone Laboratories and others and path tests conducted between Florida City, Florida, and Guanabo, Cuba demonstrated the feasibility of a system having a 20-megacycle band width and employing "beyond-the-horizon" radio transmission. "Forward tropospheric scatter" is another term used for this type of transmission. There is at present no agreement on the physical and mathematical theory of this form of transmission over long non-optical paths. All design is empirical and results in high powers, high-gain antennas and other refinements to obtain reliable and high quality transmission.

The overall system and equipment design will be discussed in the technical papers: "The Miami-Havana Radio System and its Integration Into The Telephone Networks," K. P. Stiles, E. T. Fruhner, and W. D. Siddall; and "Wide-Band UHF Over-The-Horizon Equipment," R. A. Felsenheld, H. Havstad, J. T. Jatlow, D. J. Le Vine, and L. Pollack. These papers will be presented at the Winter General Meeting of the American Institute of Electrical Engineers, to be held in New York, N. Y. in February, 1958. It is anticipated that these papers will be published in the AIEE Transactions.

The Florida-Cuba system constructed during the past year is the first wide-band system and the first application of beyond-the-horizon transmission for public telephone or television service. The new facilities were first used for public telephone service on September 12, 1957. The first television program was transmitted to Cuba on September 13, 1957 and the first television program was transmitted from Cuba on September 29, 1957. The Cuban facilities were provided by Radio Corporation of Cuba, a subsidiary of International Telephone and Telegraph Corporation, and the Florida facilities by the Long Lines Department of American Telephone and Telegraph Company. The 300 km (185-mile) beyond-the-horizon path is between a hill above Guanabo, Cuba and a site in the Everglades about 10 miles southeast of Florida City, Florida. Broad-band Type TD-2 microwave relay systems are used at each end of the beyond-the-horizon system to connect with the telephone networks. 10 KW power amplifiers having 20-megacycle bandwidth, two parabolic reflector antennas 60 feet in diameter and operation with both frequency diversity and space diversity are employed. Each of the two antennas at each end of the system are used for both transmitting and receiving. Guanabo transmits on 692 and 740 megacycles and Florida City transmits on 840 and 880 megacycles.

Figure 1 shows the 300 KM beyond-the-horizon path with the connecting

TD-2 microwave links to Havana and to Miami where connections are made to the general telephone networks. The Florida City-Miami link includes a relay station with a 250-foot tower at Goulds.

Nearly all of the radiation from one of the antennas is focussed in a high power narrow beam of 60-foot diameter but owing to phasing, dispersion and other effects this becomes something like a 1^{o} cone. Since space diversity reception is to be used at both ends of the system, each antenna is to be aimed in azimuth at the midpoint between focal centers for the two antennas at the distant end. All tests to date have indicated that best results are obtained when the antennas are aimed in the vertical plane at the effective radio frequency horizon. At Guanabo where the antennas are on a hill with no obstructions between the antennas and the sea, the antennas are aimed at the effective radio frequency horizon which is slightly above the visible sea horizon. The difference results from the difference in refractive indices for radio and light waves. At Florida City the horizon is obstructed by low mangrove and other trees on the glade and by trees and possible future construction along U. S. 1, several miles in front of the antennas. To clear these obstructions, with account being taken of earth curvature and radio wave refraction, the focal centers were elevated to 51.542 feet MSL and the antennas were aimed horizontally (Zenith angle 90^{o}).

Figure 2 is a photograph of the Florida City station looking north from the flooded glade in front of the southeast reflector antenna. On the right and left are the 60' parabolic reflectors with their associated feedhorn towers. The waveguides from the transmitters in the operating building are supported by steel frames mounted on drilled pilings and are enclosed in steel casings to protect them from hurricane debris. Interlocked protective fences are provided around the antennas to deny access to areas where the ratio frequency energy level is sufficiently high to cause damage to eyes and other human organs. The 250-foot tower supporting the TD-2 microwave antenna directed to the Goulds relay station appears to the left of center on the photograph. The white limestone rock fill is visible above the dark glade water.

Although the antenna beams can be deflected about 1^{o} from their constructed aiming, by shifting the feed horn away from the true focal center of the parabolic reflector, some loss is incurred by doing this and there are practical difficulties in observing the results of such a shift. The shift cannot be made quickly and many A-B tests over a long period of time would be required by reason of normal variations over the 185-mile path. The antennas are expensive, large, heavy structures designed to resist hurricane winds. They are steel structures with an unperforated parabolic reflecting face, similar in appearance to a drive-in theatre screen. At Florida City the antennas are constructed over the glades using drilled caisson piles extending some 35 feet into the Tamiami limestone formation.

Figure 3 illustrates the beyond-the-horizon path geometry. This figure is drawn on a 4/3-earth-radius projection which yields a straight line for the refracted radio wave, a device useful in the study of Freznel Zone clearances for line-of-sight microwave paths. In beyond-the-horizon radio propagation, some part of the energy is scattered in all directions, but more predominantly in the forward direction, presumably by discontinuities in the Troposphere.

Considering all factors, it seemed desirable to place great reliance on geodetic surveys for the control of the aiming of the beyond-the-horizon antennas at the two ends of the system. Long experience at other overseas radio stations in the United States has demonstrated the usefulness and ultimate

economy of geodetic control, not only for the aiming of directive antennas but also as a means of obtaining State Plane Coordinates and efficient effective control of all boundary, topographic and construction surveys and records. For this relatively short beyond-the-horizon path with highly directive antennas, as compared with long paths and moderately directive antennas for the high-frequency overseas systems, a greater degree of coordination was required between the control surveys at the two ends of the system. This was effected by the discussion of basic objectives and the interchange of preliminary, interim and final geographic positions and inverse position solutions, in planning conferences held in New York, Florida City and Havana, and by telephone and correspondence as the work progressed. During the course of the planning it was necessary to shift the Florida site several miles when it proved infeasible to purchase sufficient land at the preferred location along the west side of the US 1 highway. This required a slight reorientation of the Guanabo antennas about the center between feed-horn towers. Other small changes were required in the early planning to compensate for minor relocations of plant at one end or the other.

For the Guanabo site, Radio Corporation of Cuba engaged Saenz-Cancio-Martin, Engineers, as consultants for all surveying, building design and construction inspection. The geodetic control at Guanabo was handled by Ing. Antonio Aguilar, Director Topographic Division. Control for the Florida City site was handled by Max O. Laird, Overseas Equipment Engineer, American Telephone and Telegraph Company.

Geodetic Control for Florida City

Planning and Recovery

Geographic position and azimuth at the Florida City site were obtained by 2nd-order triangulation expansion from a 1st-order station and azimuth mark of the U.S. Coast and Geodetic Survey. Several towers at least 60 feet high would have been required to use other stations of the USC&GS. The expansion figures are shown in Figure 4. Checks on the recovery of undisturbed position and azimuth comprised the examination of monuments and astronomic azimuth checks.

Station QUARRY is in the glade about 95 feet east of Card Sound Road and is a firm PCC monument founded on the Miami Oolite Rock about 30" below the glade. Two reference marks for QUARRY are in PCC monuments in the rock road fill on the northeast side of Card Sound Road. There is no evidence that the reference monuments have been struck by vehicles but recovery measurements indicate slight movements away from the pavement centerline, since the USC&GS reference measurements were made in 1935. This is a result judged to be characteristic of traffic compression settlement of the rock fill of roads on this terrain. The USC&GS Azimuth Mark for Quarry also is in a PCC monument in the rock fill on the northeast side of the road. There is no evidence that it has been struck by vehicles but it must be regarded as subject to possible displacement away from the road centerline. A displacement of the order indicated at the reference marks would result in a change of about -1 second in azimuth. Three astronomic azimuth checks made at TELCO-1 yield values of +0.7, + 0.1, and + 2.6 seconds with respect to the value determined from the triangulation. (The sense of these differences is opposite to that which would result from movement of the Azimuth Mark away

from the road centerline). The astronomic azimuth checks must be regarded as subject to several seconds of systematic error resulting from undetermined deflection of the vertical affecting the leveling of the theodolite.

State plane coordinates from the U.S. Corps of Engineers 3rd-order traverse-leveling to their monument FCE-1085 differ by -0.30 ft. in X and +1.16 ft. in Y from the values derived from the triangulation expansion. Whether the Corps of Engineers' plane coordinates for FCE-1085 are within 3rd-order limits for their traverse is not determinable from the data currently at hand and these small differences are temporarily unreconciled. Use of FCE-1085 avoided the expense of constructing another monument and provides a tie between USC&GS and USED work in this vicinity.

The recovery of geographic position and azimuth at QUARRY is regarded as being satisfactory for 2-order expansion purposes.

Reconnaissance & Preliminary Solution

An on-the-job preliminary solution of the control net for geographic position and azimuth were required to permit placing rock fill and constructing the transmitter building with its rear wall nearly perpendicular to the direction toward Guanabo. This was desirable to accommodate the large rigid waveguides between transmitters and antennas and to permit drilling the caisson piles for the antennas and waveguides. For this purpose only, the base Quarry to Azimuth Mark (about 3000 feet) was measured by the subtense method using 6 positions on a Wild T-2 theodolite and a subtense base of about 25 feet across Card Sound Road. The reconnaissance triangulation comprised 1-position directions on the Wild T-2 with all triangles concluded, that is, with only two angles in each triangle being measured. Daytime observations and temporary signals at intended monument locations were used for this reconnaissance work. Triangles were computed in the field by Iris Laird, as the field work proceeded. These field computations were made in a station wagon parked along Card Sound Road, using logarithms and mosquito dope, while dozers, cranes and drilling rigs were traveling to the site. As determined by the final results the errors in this reconnaissance and preliminary solution were about 1 part in 3000 for distance and 2.9 seconds in azimuth for the aiming of the antennas, a result which is probably fortuitous. The production of this preliminary solution permitted construction work and boundary surveys to proceed while the final triangulation work was being carried out.

Local Base Line

All distances in the triangulation expansion were determined from the line Quarry to Azimuth Mark which was measured on an offset or broken base along the northeast side of Card Sound Road from the Azimuth mark to a corner within a tape length of Quarry. A 100-foot steel tape, tape thermometers and tension handle, all calibrated by the National Bureau of Standards, and second-order procedure were used for taping between 2x4 stakes with copper scribing strips. The stakes were driven solidly in the rock fill and cut to level grade. The mean length of the base from forward and backward measures was 3079.052 feet. The difference between forward and backward measures on the main tangent of 3031.834 feet was 0.018 feet, corresponding to a probable field error in the mean value of 1 part in 500,000. The unknown errors involved in the standardized constants of the taping equipment are regarded as negligible in relation to the field errors.

Monuments, Instrument and Observing Stands, and Signals

The station site is largely on the southwesterly side of Card Sound Road about 9.5 miles southeast of Florida City and 2.2 miles from Card Sound. It is salt glade with an elevation of about + 1 foot MSL and a total chloride content of about 20,000 parts per million. Here the Miami Oolite rock is covered with about 30" of soft marl, thin sawgrass, scattered small mangrove and heads (clumps) of larger mangrove, willow, alder and Australian pine. Water stands at about the mean glade level during the dry season (February) and reaches a quasi-maximum of about 2 feet above the glade in the summer and fall. Hurricanes may drive water to levels around 8 or 9 feet above mean sea level. Travel on foot is difficult and substantial use was made of aerial photographs in planning and reconnaissance to obtain clear lines for the triangulation. Some cutting was required, particularly through Australian pine on the line Quarry to FCE-1085, through a mangrove head on the line Telco-1 to Telco-3 and on the aiming lines for the two antennas. Swamp buggies were used during the final triangulation work. (One buggy is still bogged down near Telco-3).

The station marks are standard 90 mm bronze triangulation station disks cast with the legend "Geodetic Survey Station, Amer. Tel. & Tel. Co., Overseas Division." The name of the station and the year 1956 is stamped on each disk. TELCO-1, TELCO-2, and TELCO-3 have underground marks consisting of a standard disk grouted in the underlying Miami Oolite rock. The underground mark was set by the use of a 30" length of 10"-diameter furnace stack used as an open caisson. A salt-water drillers mud (Zeogel) was used to dredge the muck and seal off the water at the top of the rock. After plumbing a centering jig over the underground mark, a thin parting strip was laid over the underground mark and a concrete base for the monument was poured inside the 10" form. A 6" x 4' vitrified clay sewer tile was then set bell-end down and coaxial with the underground mark. This was filled with concrete and the surface mark was set in its top and centered vertically over the underground mark. The 10" furnace stack was then pulled for reuse at the next monument to be constructed.

Three 3 1/4" x 4 1/4" x 10' undrilled creosoted cross-arms were cut and pointed to provide 3 stakes 4 1/2' long and 3 stakes 5 1/2' long. The 4 1/2' stakes were driven solidly into the rock 2.5' from the mark and on directions 0°, 120° and 240° and connected with 2x4s nailed around the top of the sides of the triangle. Diagonal 2x4 braces were added to make this a firm instrument stand for a theodolite or transit on its tripod. The independent observers stand was constructed with the 5 1/2' stakes driven into the rock 2.5' from the mark on directions 60°, 180° and 300° with 2x4 side ties and diagonal bracing and with a 2x6 plank walkway over the stakes. Following the completion of all direction observations at the station this triangular observers platform planking was painted with white deck paint so that it will show as an identified photo control point in any future aerial photography of the area. Low level photography was flown at 2100 feet on September 9, 1957, to supplement available high-level 9-lens photography for mapping work.

A guyed 2" x 2" x 12' signal mast, painted white with black bands from the 4 to 6 and from the 8 to 10 foot levels was centered over the mark on a headless 16 d nail set point down in the mark center. A 6-volt miniature lamp with soldered leads was centered in the top of the signal mast and was used as the mark for all triangulation work. Lamp leads were extended down the

mast. Portable lantern batteries were used and will operate the lamp for about 40 hours. Signals were plumbed using two transits in the planes of the quadrature guys. The signals are left centered at all times when the station is not occupied with the instrument. To remove a signal it is necessary to remove only one guy. Lamps turned on at dusk, on nights when directions were observed, were left burning until the batteries were exhausted, batteries being less expensive than travel over the glade.

TELCO-1 ECCENTRIC has underground and surface marks set in concrete in the rock fill around the building and antennas. It cannot be occupied and reflects only an abandoned location for TELCO-1, the abandonment being made necessary by changes in site development plans which placed plant close to the monument.

FCE-1085 is a recovered 3rd-order traverse-leveling monument of the U. S. Corps of Engineers and comprises a CE 90 mm disk set in the top of a concrete filled 5" pipe projecting 2' above the glade. There is a concrete filled nail keg around the 5" pipe, the top of the keg being about 2" above the glade. The monument is solid, shows no evidence of disturbance and it is guessed that the pipe was driven into the underlying rock. Instrument and observers stands and a signal were constructed here as described in the preceding paragraphs.

At USC&GS station QUARRY, instrument and observers stands and a signal were constructed as previously described. Only a signal was required at the AZIMUTH MARK for Quarry.

The AIMING MARKs for the east and west antennas comprise guyed signals as described previously, flying white pennants and set in 2x4 stobs driven in the top of 2"x8' galvanized steel pipes which were driven about 18" into the Miami Oolite rock. These aiming marks are about 2000 feet in front of the antennas, on the exact line to Guanabo. They are equipped with lamps for use if night observations are required to avoid severe refraction over the glade, a condition which may be encountered under high water conditions.

Triangulation

All lines were observed in both directions using 6 positions of the circle of a Wild T2 theodolite. Ordinarily 8 positions are specified for 2nd-order work but experience has indicated that 6 positions are sufficient for this particular type of work. The Wild T2 is a Swiss direction theodolite, its glass circle being read with an optical micrometer graduated to seconds and estimated to 0.1 second. The direction to each signal is observed with the telescope direct and reversed to each of 6 settings of the circle spaced uniformly about 360°. All directions in the control triangulation were observed at night when conditions are more favorable. Supplemental 3rd-order work on directions to boundary monuments was done during the daytime with the instrument shaded by an umbrella. On this small-figure work careful attention is required to obtain accurate centering and leveling, firm instrument and signal settings and good intersections of the signals. If these precautions are taken the effects of systematic errors are balanced out in the procedure. The results reflect careful skilled work by the observer, Richard Davis, and good accuracy built into the Wild T2 theodolite.

The following is a summary of the results obtained:

Directions Observed (12 lines x 2 directions x 6 positions =)	144
Directions Rejected (more than 5" from mean)	3

Maximum triangle closure error	5."568
Average triangle closure error	2."473
Probable error of an observed direction	0."405
Probable error of an adjusted angle	0."159
Probable error of the weakest line, Telco-1 to Telco-3	1.8 millimeters
" " " " " " " "	1 part in 315,000

(total probable error base meas + triangulation).

The results are within 2nd-order limits and no need for greater accuracy can be foreseen. It is noted that control work of this character is done only once during the long life of the radio station and should be more accurate than any subsidiary survey which may be required during the life of the station. The land boundary survey, the aiming of antennas, the staking of all plant and all mapping and records are controlled from this initial geodetic survey.

Adjustment and Computation

All figures were adjusted by the method of least squares and the sides of the triangles were computed from the base line Quarry to Azimuth Mark. Strength of figure and the probable error of each distance were computed. Geographic positions and azimuths were computed from the adjusted triangulation using 1st-order geodetic formulas and tables for machine computation on the North American Datum of 1927 (Clarke Spheroid of 1866). Azimuth, back azimuth and distance between the Guanabo and Florida City midpoints were derived from a 1st-order inverse position computation. The midpoint at each end is equidistant (about 112.5 feet) from the focal centers for the two 60' parabolic reflectors at that end. The angle of convergence required for directing the spaced antennas at the distant midpoint was computed by the subtense method. Machine computations and ordinary geodetic procedures involving independent double determinations of each final value were employed as a check against blunders. Azimuths for the aiming of the Guanabo antennas were computed by Ing. Aguilar, using 3rd-order logarithmic formulas, and differ by less than 1/4 second from the back azimuths of the 1st-order machine computations for Florida City.

Construction Staking

Concrete monuments directly under the focal centers, aiming marks about 2000 feet in front of the focal centers and other points were set to control the location of foundation bolts and the erection of the antennas. This work was complicated by drilling, excavating, storage, cranes, concrete transit mix trucks and other construction activities on the small area of rock fill at the site. Final measurements and adjustments of the parabolic surface were made by taping from a jig locating the focal center of each antenna. Maximum permissible unadjusted deviation from the true paraboloid is 3/4" and final adjusted values of something less than 1/8" are desirable. Other tolerances involved in the structure are 1/16", 1/8" and 1/4". As-built checks show that all limits were met.

Vertical Control

Vertical control was obtained by 2nd-order double-run spirit leveling from the Corps of Engineers 3rd-order bench mark FCE-1085 to Telco-1 and to Telco-1 Eccentric. The floor of the operating building is at 9.11 ft. MSL. A

small area around the building and antennas was stripped to the Miami Oolite rock and filled with rock to about 5.5 ft. MSL. The crown of the Card Sound road varies from about 4.5 to 5.5 ft. MSL.

State Plane Coordinates

Although results in terms of geodetic data are appropriate for the aiming of the beyond-the-horizon antennas they are not generally usable by the ordinary land or construction surveyor. Accordingly, State plane coordinates for the Florida East Zone (Transverse Mercator Projection) were computed from the geodetic data and were furnished to the land surveyor, the Central Area of Long Lines and to the Southern Bell Telephone Company. Land boundary monuments were tied to the geodetic monuments by 3rd-order triangulation and the land surveyors final results are in terms of the State Plane Coordinate System. On the State Plane Coordinate Systems, geodetic control is effected by small systematic changes in scale and deviations between true and grid north to fit the spheroidal earth. This permits the surveyor to use ordinary plane surveying methods with the added advantages that his work is mutually consistent with all other surveys on the system and his plane coordinate points can be reestablished with corresponding accuracy even though all his monuments are lost.

Survey Parties

Reconnaissance, planning, specification, adjustment, geodetic computing and construction control work was done by Max O. Laird, PE&LS, Overseas Equipment Engineer, American Telephone and Telegraph Company. The base line and triangulation direction work was done by Walter A. McElfresh, PE&LS, Fort Lauderdale, Florida, with Richard Davis as Chief of Party and observer. The land boundary survey was made by Harry C. Schwebke & Associates, Miami, Florida, with A. R. Toussaint, LS in charge. All survey work for the Guanabo, Cuba, site was done under the direction of Ing. Antonio Aguilar of Saenz-Cancio-Martin, Engineers, Havana, Cuba, consultants to Radio Corporation of Cuba.

Geodetic Control for Guanabo, Cuba

General Description of the Guanabo Station

The Guanabo station is located on a hill providing direct line of sight for the TD-2 microwave path to the Havana terminal on the roof of the Masonic Temple. The parabolic reflector antennas directed toward Florida City are located on the brow of the hill overlooking the town of Guanabo, with no obstructions between the site and the sea horizon. Anchor piles in cavernous limestone were used for the parabolic antenna and microwave tower foundations and for anchors on guys to the microwave tower.

The over-the-horizon antennas are identical to those at Florida City, except for the omission of elevating 10-foot base sections not required on the hill at Guanabo. A hurricane wind load of 100 pounds per square foot was assumed in the design specification.

Existing Triangulation and Leveling

Cuba has triangulation networks established jointly by Instituto de

Cartografia y Catastro and Inter-American Geodetic Survey. The triangulation of the 1st order is practically complete, as well as part of the 2nd order and 3rd order, but they have only been adjusted in a preliminary way; so that the geodetic position of the stations are established within the approximate limit of 0.01" of latitude or longitude.

These triangulation stations consist of concrete monuments with bronze plates having dimensions and characteristics similar to those used by the U. S. Coast and Geodetic Survey. The triangulation network is referred to Clarke's Spheroid of 1866.

The same institutions mentioned have also established bench marks for vertical control referred to mean sea level, the monuments being similar to those used for the triangulation stations. This vertical control is adjusted and subject to an error of 0.0001 meter.

Station MORA and station BARRERAS near the plant site, both in the triangulation system, served to establish the position and orientation of the plant, since the position of MORA and the azimuth of the line MORA-BARRERAS were known.

A vertical control point H-184 located close to the southern side of the highway which forms the North boundary line of the plant site, was used to establish the vertical control for the plant. H-184 is 27.689 meters above mean sea level.

Accuracy of Control

An assumed error in locating the position of the midpoint of the focal centers of the parabolic reflector antennas of .01" in latitude or longitude would be the equivalent of a mean displacement of 0.30 m. at the latitude and longitude of Guanabo. This displacement, taking into consideration the distance between Guanabo and Florida City, approximately 300 kilometers, would produce an error in the azimuth between these points of less than 0.01", which is much less than may result from practical imperfections in the construction of the structural elements of the parabolic reflectors; so it was decided that an error of 0.01" in Latitude or Longitude was sufficiently small from a practical point of view.

Monuments

Monuments were used to establish the control points in a permanent way. The monuments were slightly smaller than the type used by the U. S. Coast and Geodetic Survey. All the control points established for the development of the control and mapping had the shape of a truncated pyramid, while those used for establishing the position and orientation of the antennas had the shape of a truncated cone.

The reinforced concrete monuments were placed so as to project 0.10 meter above ground level with a depth of 0.30 meter or more depending on the nature of the ground and with a square or circular cross-section of 0.20 meter on the side or diameter. A cast solid bronze plate shaped like a section of a sphere, with a roughened shank to assure good bond, is embedded in the center of the top of the monument, on which the proper inscription can be made.

Triangulation Expansion

Geodetic position and azimuth were established for the plant site at

Guanabo by expansion triangulation from station "MORA," and measuring one side of a quadrilateral as a base. The azimuth was taken from the known azimuth of the line "MORA-BARRERAS." A monument was established at each vertex of the quarilateral and numbered 1601, 1620, 1621. The fourth vertex was station "MORA." The base was measured along "MORA"—1620. Station 1601 is located within the property of the Radio Corporation of Cuba. The expansion figures are shown in Figure 5.

Base Line

Stakes were driven along the base line with copper strips on top for measuring marks, and a line of levels was run to establish elevations with an error of 0.001 meter. A 50 meter steel tape was used for the actual measurement, with a tension of nine Kg., in accordance with the instructions of the manufacturers, and thermometers were used for the temperature corrections. Elevation above mean sea level was known for Station "MORA." After applying suitable corrections for temperature, catenary, difference of elevation and reduction to sea level the base line was found to be 320.037 $\pm$ 0.001 meters.

Directions

All directions were observed with a Kern type DK2 1-minute direction theodolite making eight series of observations in each direction. All observations were made during the day using an umbrella, which was satisfactory for the precision required. The signals were range poles made of aluminum pipe, painted red and white, and supported on steel tripods. Refraction was negligible due to the height of the line of sight above the ground and the excellent telescope of the instrument used. The maximum error of closure was found to be 6" with a mean value of 4".

Adjustment

Adjustment of the observed values of the angles and distances of the quadrilateral was made by the method of least squares, the results being as follows:

Line "MORA" - 1601: distance 727.630 m $\pm$ 0.001 m
azimuth 161° 24' 34" $\pm$ 1"

Computations for Station 1601 and Azimuth of Line 1601 - "MORA"

Knowing the geodetic position of "MORA" and the length and azimuth of line MORA - 1601, the position of 1601 and azimuth of the line 1601 - MORA was calculated by using form No. 27 of the U. S. Coast and Geodetic Survey, checked by the calculation of X and Y rectangular coordinates for true North, taking into consideration that one second of latitude corresponds to 30.761 m and one second of longitude corresponds to 28.447 m at the mean position of the control—stations MORA and 1601.

Results:

1601: Lat 23° 09' 46."89 $\pm$ 0."01 North
Long 82° 09' 18."28 $\pm$ 0."01 West

1601 - MORA: Azimuth 16° 24' 34" $\pm$ 1" from the North
All referred to Clark's Spheroid of 1866

Topographic Survey

Control monuments were established, similar to those used for triangulation, based on cartesian coordinates as a basis for the topographic details needed later in the layout of construction and the installation of the plant itself, within the property of the Radio Corporation of Cuba.

The property of the Radio Corporation is bounded; on the North by the southern limit of 50 meter wide expropiated zone, which separates it from the road between Guanabo and Habana known as VIA BLANCA, on the South by the northern limit of the 15 m expropiated zone which adjoins "Segundo Boulevard" on the East by Dr. Andreu's property and on the West by the development known as Alturas de Marbella.

The distances between vertices of the traverse polygons were measured with a 50 meter steel tape, keeping the tape horizontal with a steady pull. The measurement of the sides of the polygon had an error of 0.01 m; sufficient accuracy for the topographic work.

The lengths of 1603 - 1623 and 1605 - 1622 were determined by the subtense method. Perpendiculars were erected on both sides of the polygon side. Stations on them were established at 10 m from the side of the polygon, thus forming two triangles with a base of 10 m and whose common altitude was the side of the polygon the length of which was to be computed. The three angles of each triangle were measured and the common altitude was adjusted with an error of 0.01 m.

A Kern DK2 direction theodolite was used with four positions or series of observations for each direction. An umbrella was not required for these daytime observations.

The vertices of the boundary lines of the property and the details of topography were determined by observing directions and measuring distances from the vertices of the control polygon with an error of 0.1' in angle and 0.01 meter in distance.

Plane Coordinates

Local plane coordinates were used for reference to the control points, boundary lines and details of topography, with Y axis parallel to the true North and selecting the origin to make the coordinates of vertex 1601: X = 150.00, Y = 150.00. Curvature was disregarded as the maximum distance was 220 m on the X or Y axis.

The error in measuring the angles of the polygons was 0.'7 which gives a relative value of 1:11,000. The probable error of position in control X or Y is 0.01 meter. Coordinates of the control points were adjusted by the Bowditch method.

The vertices of the boundaries were calculated from the control points, and then the length of the sides, the azimuth of each line and the area were determined.

Vertical Control

Vertical control point H-184 with an elevation of 27.689 meters above mean sea level and established by I.C.C. and I.A.G.S., is located on the southwest wing wall of the culvert on Via Blanca about 10 meters from reference monument 1605, and was used as the starting and terminal point of the line of levels run along the length of the polygon between monuments.

The elevations of the control points were established as indicated above with an error of 0.001 meter, thus establishing coordinate "Z" with reference to the horizontal plane.

Several lines of levels were run approximately parallel to each other on each side of the control polygon: Initial and final readings were taken on a reference monument with an error of closure of 0.01 meter. With these lines of levels it was possible to layout contour lines on the map, to guide the location of the plant structures.

A line of levels was run along the Axis of the Via Blanca and also of Segundo Boulevard, taking readings on either side of their axes with an error of closure of 0.001 meter, which is well within the limits for ordinary construction of paved areas.

Location and Orientation of Plant at Guanabo

After mapping the station property the most favorable location for the plant building was chosen jointly by the American Telephone and Telegraph Company and the Radio Corporation of Cuba and the location was indicated on the drawing made.

The position of the building was calculated by reference to the column axes, placing the front of the building approximately perpendicular to the imaginary line joining Guanabo and Florida City.

The position of the building determined the position of the waveguide outlet. It was then necessary to determine the position of the midpoint between the focal centers of the parabolic reflector antennas, which was done after an interview between the Engineers of the American Telephone and Telegraph Company and GEOTOPO.

Midpoint K was established 7.620 meters to the North of column axis 4 and 3.036 meters to the East of column axis C. This made possible the determination of the X and Y as well as the geodetic coordinates of K.

With the geodetic coordinates of K, and the corresponding values of the midpoint at Florida City, the geodetic azimuth of the line joining these points was calculated with a variation of 41 minutes compared to the assumed value for the building.

After establishing the geodetic coordinates of the focal centers E and W, it was necessary to calculate the aximuth of line E-K Florida City and W-K Florida City, which are the lines for aiming the axes of the parabolic reflector antennas in the horizontal plane.

The inclination of the elevated antennas from the vertical required to direct the radio beams at the effective radio frequency horizon was calculated using the normal refractive index for the radio frequencies and the antennas were aimed to this value, which is slightly above the visible horizon, owing to differences in the refractive indices for radio and for light waves.

Construction Monuments

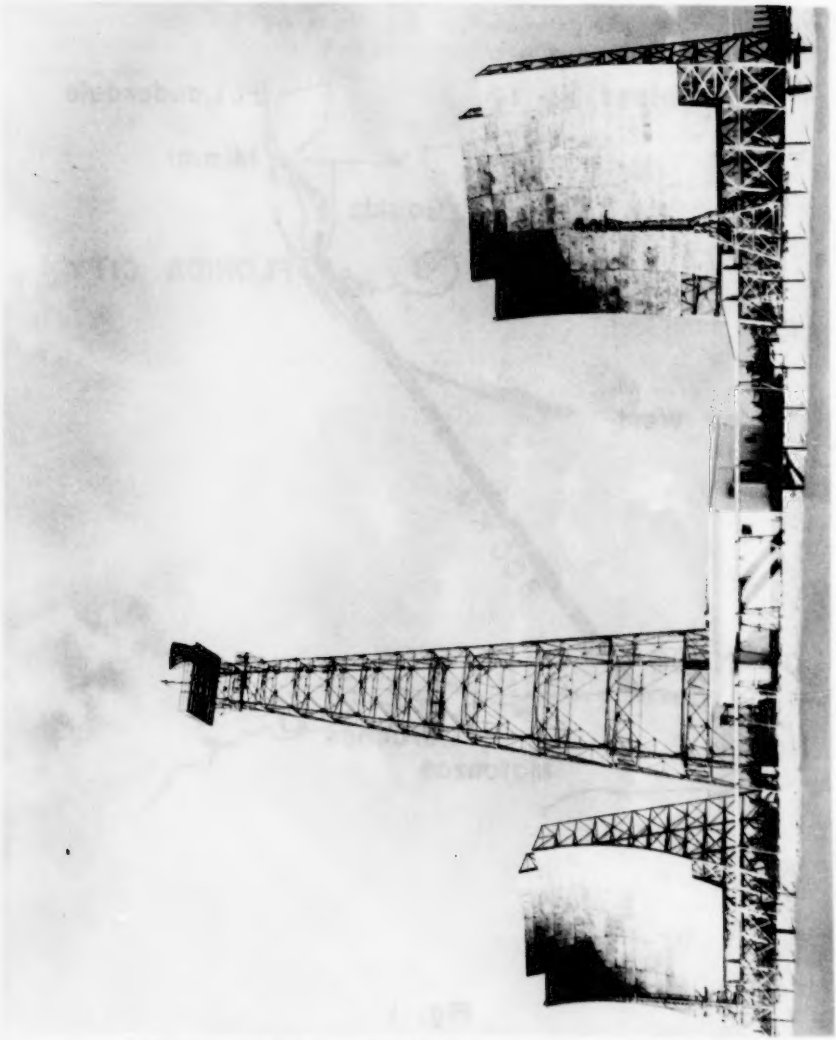
Control monuments of the type previously described were established in order to locate on the ground the line E-W, point K and aiming line K Florida City for each one of the parabolic reflector antennas, and to facilitate the layout of the towers and antennas. Auxiliary lines and points were also established, based on the above and the dimensions of the structural members to assure the correct and proper erection of the towers and antennas.

All the monuments placed to establish these lines and points were checked

in position by means of coordinates with reference to control monuments 1601, 1602, 1607 and 1608.

The reference points required to erect the towers and antennas and to check this work after erection, were done by direct sights with the monuments previously established for that purpose; eliminating the need to measure any angle or distance; for the purpose of accelerating the construction and erection work of these units.

Fig. 1
BEYOND-THE-HORIZON PATH WITH CONNECTING MICROWAVE LINKS



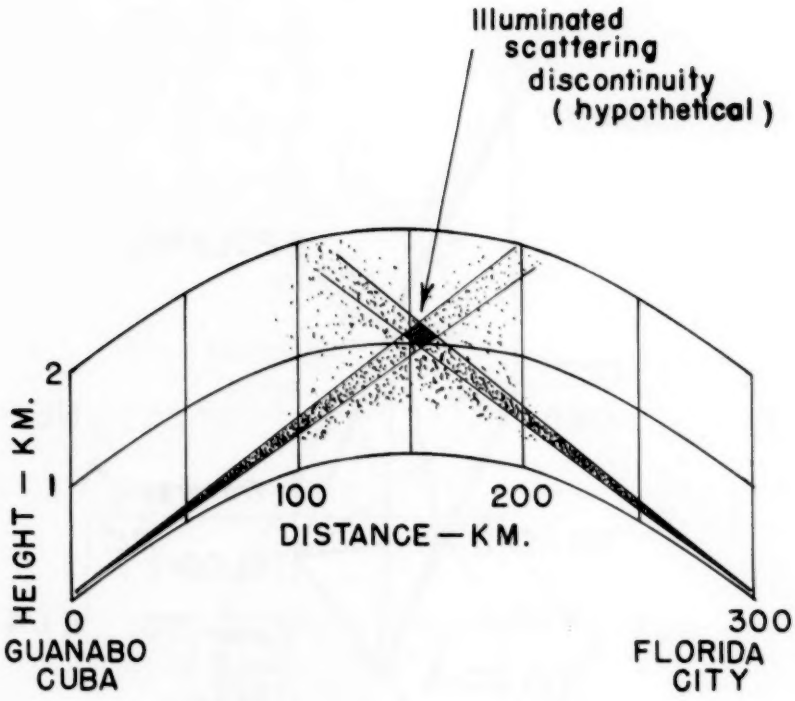


Fig. 3
BEYOND-THE-HORIZON
PATH GEOMETRY
(4/3 EARTH RADIUS)

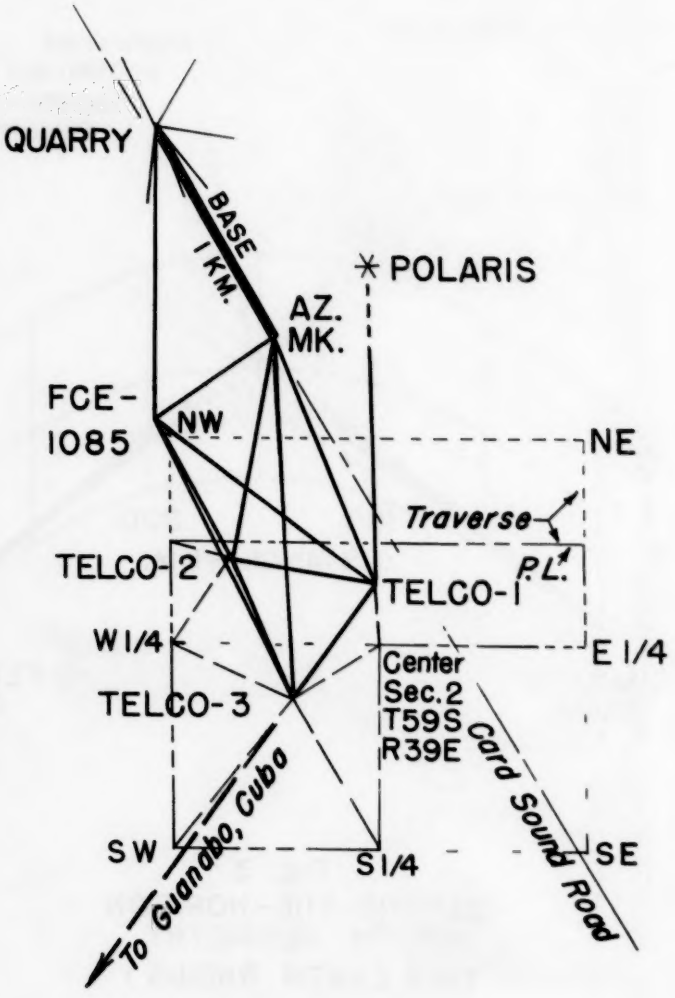


Fig. 4
HORIZONTAL CONTROL
FLORIDA CITY

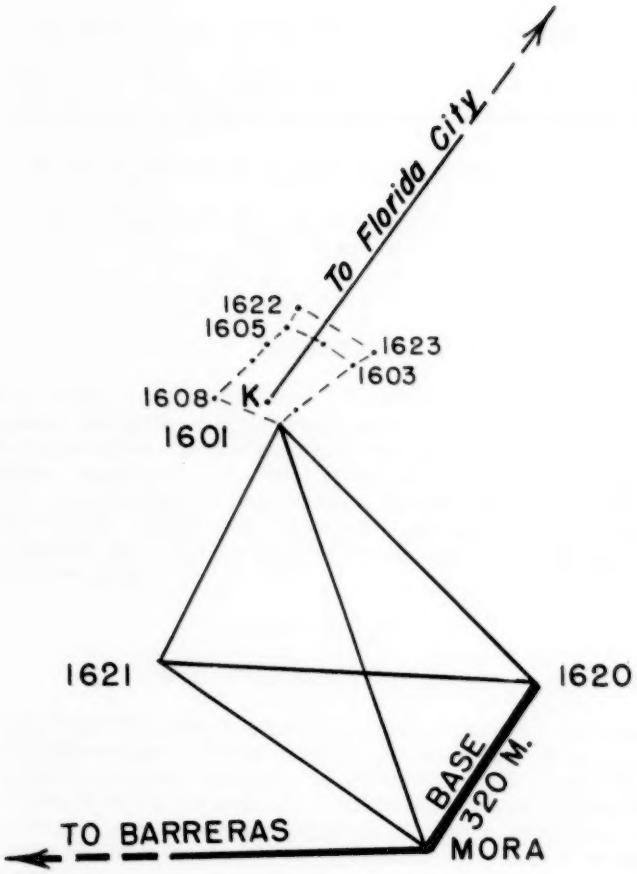


Fig. 5
HORIZONTAL CONTROL
GUANABO, CUBA

Paper 1595 April, 1958 SU 1

Journal of the
SURVEYING AND MAPPING DIVISION
Proceedings of the American Society of Civil Engineers

SHORT METHODS IN ADJUSTMENT OF OBSERVATIONS

M. V. Smirnoff[1] and Paul E. Wylie,[2] Members, ASCE
(Proc. Paper 1595)

SYNOPSIS

In the process of adjusting observed measurements by the method of least squares, the question often arises as to the validity of various "short cuts" in the particular circumstances. These short cuts are rules, devised to eliminate a large part of the labor of calculation that is involved in formal application of the method of least squares. In this paper, the writers discuss such rules and their applicability to observations which should determine either a straight line, or a curve which can be reduced to a straight line in its logarithmic form.

Observations with Two Variables

If a set of observed values depends upon two variables, these observations may be plotted relative to a pair of coordinate axes, thus giving a certain scatter of plotted points. If there is no known law which a given set of observed values must follow, such a law may be approximated from the shape of the curve that best fits the plotted points. If, on the other hand, there is a known law of variation which may be expressed in the form of an equation, then the problem is to find a set of constants in such an equation, so that the distances of plotted points from the curve, along the ordinates, will represent the most probable values of the respective errors in observed values. According to the principle of least squares, the sum of the squares of these errors must be a minimum. If both variables are subject to error, and hence require adjustment, then the sum of the squares of errors along both axes must be a minimum. If, however, only one variable requires adjustment, then

Note: Discussion open until September 1, 1958. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. Paper 1595 is part of the copyrighted Journal of the Surveying and Mapping Division, Proceedings of the American Society of Civil Engineers, Vol. 84, No. SU 1, April, 1958.

1. Associate Prof. of Civ. Eng., North Carolina State College, Raleigh, N. C.
2. Retired. Formerly Lecturer, Dept. of Eng., Univ. of California, Los Angeles, Calif.

the sum of the squares of errors along the axis of that variable only will have to be a minimum. A common example of the latter case appears when one of the variables is the time of observation. Certainly the time of observation should not be changed because of an error in the quantity observed.

Fig. 1 shows a segment MN of a straight line to which a plotted value of an observation, represented by the point c, must conform. If the error is along the X axis only, then the line ac is the error; if the error is along the Y axis only, then the line bc is the error; but if the errors are equally probable along both axes, then the line dc, perpendicular to mn, must be the error, because it represents the shortest distance to the line mn. The sum of the squares of such distances will be a minimum for a given set of observations.

In the example considered in Fig. 1, the variables were supposed to conform to a law which was expressed, graphically, by a straight line. Many observations fall logically into this category. The discussion, however, may be extended, unchanged in principle, to any curve which will plot as a straight line in its logarithmic form. This includes any equation of the form $y = ax^n$, where a is any real constant, and n can be any positive or negative real exponent.

If the observed values requiring adjustment are of different weights, then the various points will influence the fitting of a curve in proportion to their respective weights. For example, if one observation carries twice as much weight as another, it could be represented by a double point. This would require an extra coordinate for the point, relative to each axis, regardless whether only one or both coordinates require adjustment. Thus, in any case involving weighted observations, both the X and Y coordinates of each observation must be multiplied by the weight of the observation involved. This must be done before any other operations are undertaken.

The Least-Squares Line Must Pass Through the Center of Gravity of the Plotted Points

Let it be given that $v_1, v_2, v_3, \ldots, v_n$ represent respectively, the errors of points 1, 2, 3, . . . , n. Regardless of whether errors occur along one coordinate direction, or both, $v_1^2 + v_2^2 + v_3^2 + \ldots + v_n^2$ must be a minimum. Taking n partial derivatives with respect to each v, equating each derivative to zero and adding the results, the following well-known relation is obtained:

$$v_1 + v_2 + v_3 + \cdots + v_n = 0 \qquad (1)$$

Since the summation of distances of a group of points from a line passing through the center of gravity of the points must also be equal to zero, it is evident that the least square line must pass through the center of gravity of all plotted points.

The Least-Squares Adjustment for Points Subject to Error in One Coordinate Only

Fig. 2 shows a set of coordinate axes having its origin at the point c, which is the center of gravity of all observations. Let A be any point in the group of observations, and let the line CD be the sought least-squares line

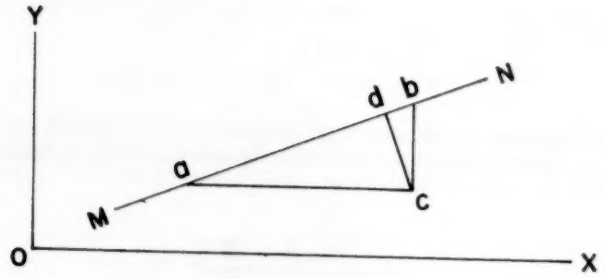


Fig. 1

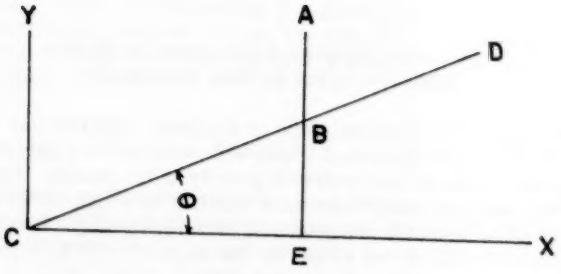


Fig. 2

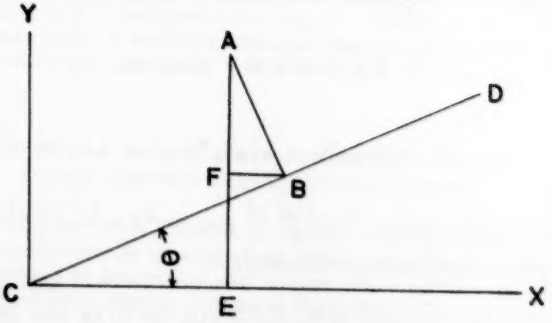


Fig. 3

having the slope $m = \tan \theta$. Assuming that all errors are in the Y coordinates, the error in the position of point A, relative to the least-squares line, must be $AB = v_a$. Since $BE = mx_a$, $v_a = y_a - mx_a$, or, for the whole set of points, $\Sigma v = \Sigma(y-mx)$. Also,

$$\Sigma v^2 = \Sigma y^2 - 2m\Sigma xy + m^2\Sigma x^2 \qquad (2)$$

Since Σv^2 must be a minimum, Eq. 2 must be differentiated with respect to m and the result equated to zero. Solving the resulting equation for $m = \tan \theta$, we find

$$\tan \theta = \frac{\Sigma xy}{\Sigma x^2} \qquad (3)$$

If the points have varying weights, w, each coordinate of each point will have to be multiplied by the weight of the point, and Eq. 3 will assume the form of Eq. 4 below.

$$\tan \theta = \frac{\Sigma w^2 xy}{\Sigma (wx)^2} \qquad (4)$$

The Least-Squares Adjustment for Points Subject to Error in Both Coordinates

Let it be required to find the slope of the least-squares line for a group of points the X and Y coordinates of which are referred to a pair of rectangular axes having the origin at the center of gravity of the points. These axes are parallel to the original respective axes relative to which the original data were plotted. Fig. 3 shows one point, A, of such a group. It also shows the least-squares line CD, whose slope, or the angle $\theta = DCX$, it is required to find.

The line AB in Fig. 3, perpendicular to CD, is the error v_a in the position of point A, due to errors AF in the Y coordinate, and FB in the X coordinate. Taking into account all plotted points, and assuming that all coordinates carry the same weight, it is required to find the value of Σv^2 that is minimum. By transformation of Y coordinates to the axis perpendicular to CD, it is found that

$$v = y \cos \theta - x \sin \theta,$$

and

$$v^2 = y^2\cos^2\theta + x^2\sin^2\theta - xy \sin 2\theta \qquad (5)$$

Substituting in Eq. 5, $\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$ and $\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$ and simplifying, the following expression is obtained:

$$v^2 = \frac{y^2 + x^2}{2} + \frac{y^2 - x^2}{2} \cos 2\theta - xy \sin 2\theta \qquad (6)$$

Differentiating Eq. 6 with respect to θ, equating the result to zero, solving

for tan 2θ, and using Σ to indicate the summation of values for all points, the following equation is found:

$$\tan 2\theta = \frac{2\Sigma xy}{\Sigma x^2 - \Sigma y^2} \qquad (7)$$

The foregoing procedure is essentially the same as that employed in engineering mechanics to find the principal axes for a given area.

If the various points are assigned different weights, w, then Eq. 7 will take the form

$$\tan 2\theta = \frac{2\Sigma w^2 xy}{\Sigma (wx)^2 - \Sigma (wy)^2} \qquad (8)$$

Eq. 7 makes possible a least-squares solution in two variables simply by substituting proper values therein, provided, that deviations are equally probable in X and in Y coordinates. Whether they are or not must be determined from the nature of the observed values. For example, the time of an observation (usually plotted as the X coordinate) may be measured with extreme precision, but the corresponding Y quantity, measured at that time, may be subject to large random errors. In such a case, it would be logical to hold the X values fixed and to adjust only Y values. Thus the use of Eq. 3 would be indicated.

Example

Let it be required to find the least-squares line for a group of 27 points having x_o and y_o coordinates as shown in Table I. The table shows also computed x and y coordinates for parallel axes with the origin at the center of gravity of the 27 given points. There are additional columns showing computed values for x^2, y^2 and xy; also, summation values at the bottom of the table.

If the data given in Table I are subject to errors only in the direction of the Y axis, then Eq. 3 must be used to find the angle θ that the least-squares line makes with the direction of the X axis. Substituting in Eq. 3, the proper values from Table I, it is computed that $\tan \theta = -0.4764$, or $\theta = -25^{\circ}28'$.

If, on the other hand, it would be assumed that the probability of errors is the same relative to each axis, Eq. 7 must be used. It gives $\tan 2\theta = -1.6519$, or $\theta = -29^{\circ}24'$.

The Method of Using Three Centers of Gravity for Fitting a Straight Line to a Group of Points

The method of using three centers of gravity of plotted points for fitting a straight line is described in several handbooks. (See, for instance, Civil Engineers' Handbook by Merriman and Wiggin, Fifth Edition, p. 1324.) This method, which hereafter will be called the "center-of-gravity method," requires one to find the center of gravity of all the points; also, the coordinates of points relative to a set of axes parallel to the original one, and having its origin at the center of gravity. So far, the procedure is the same as in the

TABLE I

Pt.	X_o	Y_o	X	Y	X^2	Y^2	XY
1	12.9	5.0	2.9	-1.0	8.41	1.00	- 2.90
2	11.7	6.8	1.7	0.8	2.89	0.64	1.36
3	11.2	4.3	1.2	-1.7	1.44	2.89	- 2.04
4	13.3	3.7	3.3	-2.3	10.89	5.29	- 7.59
5	8.1	6.3	-1.9	0.3	3.61	0.09	- 0.57
6	7.8	7.2	-2.2	1.2	4.84	1.44	- 2.64
7	9.4	6.4	-0.6	0.4	0.36	0.16	- 0.24
8	14.9	3.3	4.9	-2.7	24.01	7.29	-13.23
9	10.3	6.1	0.3	0.1	0.09	0.01	0.03
10	6.5	7.7	-3.5	1.7	12.25	2.89	- 5.95
11	5.6	8.0	-4.4	2.0	19.36	4.00	- 8.80
12	11.4	5.3	1.4	-0.7	1.96	0.49	- 0.98
13	7.4	8.0	-2.6	2.0	6.76	4.00	- 5.20
14	8.2	7.2	-1.8	1.2	3.24	1.44	- 2.16
15	12.6	5.8	2.6	-0.2	6.76	0.04	- 0.52
16	9.2	5.6	-0.8	-0.4	0.64	0.16	0.32
17	5.4	7.2	-4.6	1.2	21.16	1.44	- 5.52
18	9.3	7.7	-0.7	1.7	0.49	2.89	- 1.19
19	6.8	9.0	-3.2	3.0	10.24	9.00	- 9.60
20	8.2	8.8	-1.8	2.8	3.24	7.84	- 5.04
21	8.3	5.0	-1.7	-1.0	2.89	1.00	- 1.70
22	14.2	6.6	4.2	0.6	17.64	0.36	2.52
23	13.6	2.2	3.6	-3.8	12.96	14.44	-13.68
24	5.7	5.6	-4.3	-0.4	18.49	0.16	1.72
25	10.9	7.9	0.9	1.9	0.81	3.61	1.71
26	13.3	2.4	3.3	-3.6	10.89	12.96	-11.88
27	14.3	2.9	4.3	-3.1	18.49	9.61	-13.33
	270.5	162.0			224.81	95.14	-107.10

above described method to find the least-squares line. From here on the two methods differ. In the center-of-gravity method, the points are subdivided into two sub-groups; those having positive and those having negative Y coordinates, if the slope of the fitting line may be expected to be between 45° and 135°; or those having respective positive and negative X coordinates if the slope of the fitting line may be expected to fall between plus and minus 45°. Next, the center of gravity of each sub-group is found, and the straight line connecting the three centers of gravity is the fitting line for the given group of points.

That the line joining the three centers of gravity is a straight line may be proved as follows: Let the three center of gravity points be c_0 for the whole group, c_1 for one sub-group, and c_2 for the other sub-group. Connecting c_1 and c_0 by a straight line and designating the distances of points 1, 2, 3, etc., from this line by z_1, z_2, z_3, etc., one finds that the summation of z's for all points must be equal to zero, and the summation of z's for the first group must also be equal to zero, since the straight line passes through points c_0 and c_1. Consequently, the summation of z's for the second sub-group must also be equal to zero. If so, then the straight line must pass through the center of gravity, c_2, of the second sub-group also.

In general, the center of gravity line does not coincide with the least-squares line. Designating the two sub-groups into which the points are divided in the center of gravity method as group 1 and group 2, the coordinates of points in these respective groups by x_1, y_1 and x_2, y_2 and the number of points in each group by n_1 and n_2, then the slope, m, of the line passing through the three centers of gravity will be

$$\frac{\Sigma y_1 / \Sigma n_1}{\Sigma x_1 / \Sigma n_1} = \frac{\Sigma y_1}{\Sigma x_1};$$

or, using the coordinates of group 2, the same slope will be $\Sigma y_2 / \Sigma x_2$. Thus, for the center of gravity line to coincide with the least-squares line, this slope must be equal to the expression for $\tan \theta$ in Eq. 3, if only one coordinate of each point is assumed to be subject to error, or to $\tan \theta$ derived from Eq. 7, if both coordinates may have errors. Calling these two possibilities Case 1 and Case 2, respectively, the following necessary condition for Case 1 may be written:

$$\frac{\Sigma Y_1}{\Sigma X_1} = \frac{\Sigma Y_2}{\Sigma X_2} = \frac{\Sigma XY}{\Sigma X^2} \qquad (9)$$

Eq. 9 is valid when all plotted points fall on a straight line. This is the trivial case in which no problem exists.

At least one other case may be cited for which Eq. 9 will hold. Let it be assumed that an equal number of observations were taken at two different instants of time. Then the center of gravity of all plotted points will project on the X axis half way between the times of two observations. Thus when coordinate axes will be drawn through the center of gravity of all points, one half of X values will be positive and another half negative. Also the absolute value of all X coordinates will be equal.

If in addition it will be assumed that all Y values observed at the first instant of time will fall say in the third quadrant and those observed at the

second instant of time will fall in the first quadrant, all terms in Eq. 9 will be positive and the equation could be written in the following form:

$$\frac{\Sigma|Y_1|}{\Sigma|X_1|} - \frac{\Sigma|Y_2|}{\Sigma|X_2|} - \frac{\Sigma|XY|}{\Sigma|X^2|} \qquad (10)$$

The absolute values of X can be factored out in the numerator and the denominator of the last term and canceled out, leaving $\frac{\Sigma|Y|}{\Sigma|X|}$, which would make the equation valid for the case considered.

As a rule, however, the two methods will give different lines. The difference between the two methods may be demonstrated on the example given in Table 1.

Put all points having negative X coordinates in group 1, and those having positive X coordinates in group 2; then the slope of the line for the center of gravity method will be

$$\Sigma y_1/\Sigma x_1 = -15.7/34.1 = -0.4604.$$

This value of m corresponds to $\theta = -24^{\circ}43'$. This does not check either value of θ computed above for the least-squares line, although in this case it comes fairly close to the $-25^{\circ}28'$ value computed above for Case 1.

CONCLUSION

The Gaussian curve of distribution of errors, upon which the method of least squares is based, assumes that the errors are purely random in nature. In observational adjustment, it is necessary, therefore, to be sure that systematic errors are absent. There are, on the other hand, certain applications of the least-squares method in which it is unnecessary to consider the nature of the deviations. (One can hardly call them errors.) For instance, in finding the minimum moment of inertia of an area, it is necessary to minimize the expression $\int y^2 dA$. This operation is equivalent to finding the least-squares line through the area.

The short-cut methods considered in this paper have been reconciled with the formal least-squares theory, with the exception of the center-of-gravity method. The center-of-gravity line does not, as a rule, coincide with the least-squares line, and therefore, if one considers the least-squares line the best adjustment to the data, then the center-of-gravity method should not be used.

The field of application of the method of least-squares is as broad as the field of measurement itself. An interesting example in the field of surveying is found in Surveying for Civil Engineers, by Philip Kissam, 1956 edition, p. 628. Although the examples worked out in this textbook follow a general procedure of application of the method of least squares in order to demonstrate the method, the same examples could be worked out, and the same results obtained, by the short-cut techniques discussed in this paper. For instance, Example 1, Art. 22-8 can be worked out by the use of Eq. 3.

Paper 1605 April, 1958 SU 1

Journal of the

SURVEYING AND MAPPING DIVISION

Proceedings of the American Society of Civil Engineers

CONTENTS

DISCUSSION
(Proc. Paper 1605)

Page

Foreword to Manual on Highway and Bridge Surveys and Chapter I—State Plane Coordinates
Progress Report of the Committee on Highway and Bridge Surveys of Surveying and Mapping Division. (Proc. Paper 1306, July, 1957. Prior discussion: none. Discussion closed.)
by J. C. Carpenter 1605-3

Note: Paper 1605 is part of the copyrighted Journal of the Surveying and Mapping Division, Proceedings of the American Society of Civil Engineers, Vol. 84, No. SU 1, April, 1958.

PROGRESS REPORT OF THE COMMITTEE ON HIGHWAY AND BRIDGE SURVEYS OF THE SURVEYING AND MAPPING DIVISION:

Foreword to Manual on Highway and Bridge Surveys and Chapter I—State Plane Coordinates[a]

Discussion by J. C. Carpenter

J. C. CARPENTER,[1] M. ASCE.—The Introduction under the Foreword includes much current information that is well known to highway engineers and should not be included in a manual of this character. Use of the 1929 Mean Sea Level Datum for elevations is common practice. The Coast and Geodetic Survey and the Geological Survey have established so many bench marks that there is no excuse for use of an assumed datum. This is not true for horizontal control, principally because the stations for this control are not as well distributed. Stations on the primary geodetic network were located on high ridges for good vision whereas the control levels were run along roads and railroads. The case shown on page 12 with two geodetic stations close to the highway is the exception, rather than the rule. Unless the highway survey can close on two stations with known coordinates the accuracy of a survey cannot be checked. The primary advantage as set forth on page 8 under noted paragraphs (a), (b) and (d) cannot be realized and none of the points can be considered to be permanently located.

The use of State Standard Plane Coordinates as control for highway surveys is highly desirable. Execution of surveys with the short range objective of serving only the immediate needs of construction has never been economically defensible. Members of the committee on highway surveys have advocated use of geodetic control in personel interviews with responsible highway officials and through publicity for the past two decades. Examples of the publicity are: "Permanent Value for Highway Surveys through Geodetic Control," Civil Engineering, Vol. 5 December, 1935; "Oregon Uses Lambert Conformal Conic Projection in Highway Surveys," Civil Engineering, Vol. XV, May, 1935; "Higher Accuracy in Highway Surveys Brings 'Long Run' Economy," Civil Engineering, Vol. XXI, July, 1951. Similar publicity was inserted in Surveying and Mapping and in American Highways. It was expected that this crusade would stimulate interest in the use of geodetic control and acquaint the highway engineers with the value of accurate surveys and the use of the State Standard Plane Coordinates. Campaigns were carried on to persuade the state highway departments to show the plane coordinates on all county highway maps and reference origins and destination to the coordinates.

The response by practicing highway engineers was extremely disappointing,

---

a. Proc. Paper 1306, July, 1957.

1. Senior Highway Traffic Engr., Coverdale & Colpitts, New York, N. Y. Formerly Chairman, Committee on Highway and Bridge Surveys, ASCE.

In a few cases where the attempt was made to tie in surveys to the geodetic network it was found that the stations were on high ridges or located so far from the survey that several miles of traverse were required, and the whole idea was abandoned. It seems doubtful that highway engineers, now loaded with additional work in connection with the 1956 Federal Highway Act, will immediately adopt the use of plane coordinates and tie their surveys to the geodetic network when they have never shown interest in it before.

This does not mean that the idea must be abandoned. The emphasis should be placed on accuracy in keeping with reasonable standards for all highway surveys. If the survey can be tied to the geodetic network with reasonable facility the manual should certainly recommend it. Within recent years the United States Geological Survey has shown the Plane Coordinate grids on their topographic quadrangle sheets by means of marginal ticks. For highway surveys through areas covered by these recent surveys, the coordinates can be established with reasonable accuracy and the Geological Survey may provide the traverse notes for their control so that the highway survey may be checked for accuracy.

The "Introduction" will be much more effective and will encourage practicing highway engineers to read the rest of the manual and perhaps to apply it, if it is condensed to two concise paragraphs. The first paragraph would set forth the reasons for a manual in concise language and without reference to the previous failures to provide permanent surveys or to the passing 1956 Highway Act. The second paragraph might well stress the importance of tieing the surveys to the geodetic network where economically feasible and use of plane coordinates. Accuracy should be emphasized and re-emphasized so that when the survey may eventually be tied to the network the points will be permanently located and by continually picking up ties the value of the surveys will be firmly established.

Much of the information on plane coordinates should be omitted. The manual should include a listing of the States having Lambert projections and those using Transverse Mercator and suggest that if any engineer wishes to explore the basic theory of the Lambert he should write to the Superintendent of Documents for a copy of Special Publication 194 of the U. S. Coast and Geodetic Survey and if his state is controlled by Transverse Mercator, Special Publication 195 will provide all the details. Special Publication 235 is the best for general practical study of the application of plane coordinates.

The tables on pages 6 and 7 should be omitted. They apply to accuracy standards for geodetic work and will not be understandable to highway engineers. In place of these tables the "Standards of Accuracy for Highway Surveys" of the American Congress on Surveying and Mapping, adopted in 1949, which were carefully developed by a committee of practicing highway engineers should be fully quoted in the text.

All of the information on "Transformation of Geodetic to State Plane Coordinates" should be omitted. This procedure is no longer necessary and it will lead the highway engineer who may want to use plane coordinates to believe it is a very complicated system and discourage him in its use. The manual should indicate that the coordinates of all stations of the Coast & Geodetic Survey may be obtained from the Survey and that additional information may be obtained from the Geological Survey if they have recently surveyed the area. The application of the conversion procedure may be useful as a text for college students but it is not of any interest to a busy highway engineer or to survey party personnel. The manual should emphasize the simplicity of the

use of coordinates and that the X and Y coordinates are expressed in feet and may be understood by any land surveyor who understands latitude and departure in his simple surveys. Corrections for altitude may be mentioned but not encouraged for they may be applied when final adjustments are made and ordinarily they will fall within the limits of second order surveys.

It cannot be too strongly emphasized that the important features of any highway survey are that it should be made with accuracy to comply with standards established for highway surveys of the class under consideration, that the survey should be tied to the geodetic network where economically feasible and that surveys based on State Plane Coordinate Control are simple and do not require the use of higher mathematics.

Paper 1617 April, 1958 SU 1

Journal of the
SURVEYING AND MAPPING DIVISION
Proceedings of the American Society of Civil Engineers

THE GEODIMETER AND TELLUROMETER[a]

Austin C. Poling[1]
(Proc. Paper 1617)

Among the fundamental constants of nature, one of the most intriguing to the scientist is the speed of light, commonly denoted by the letter c. One of the earliest scientists to try to determine the speed of light was Galileo who used a hand-operated shutter to send flashes of light from a lantern to an assistant on a nearby hill, who in like manner signaled back. Although this experiment failed, many scientists since then have been engaged in finding new methods for the determination of c. The modern methods began with the toothed wheel or rotating mirror which were used as mechanical choppers of the light beam. Although these were a very great improvement on Galileo's crude method, and gave fairly accurate results, it was the Kerr cell which led to the modern accuracy in the determination of c. In 1875 John Kerr discovered that an isotropic dielectric medium placed in a strong electrical field acquires the properties of double refraction of a uniaxial crystal with its optic exis in the direction of the lines of force. This is the well known Kerr electrical-optical effect. This effect can be used as an interrupter or chopper for a light beam. A Kerr cell constructed for this purpose was first used by Karolus in 1925 in an instrument used for the determination of the speed of light. About 1950 the Kerr cell was used by Dr. Erik Bergstrand, a Swedish scientist, in his experiments with the speed of light. The modern Kerr cell used by Dr. Bergstrand consists of an optical glass cell with two electrodes immersed in a nearly pure solution of nitrobenzene, which when used with crossed polaroids and an appropriate electrical field, becomes an electronic light chopper. After Dr. Bergstrand had found a good value for the speed of light, over precisely measured distances, with his newly devised equipment, he decided to reverse the process and holding the speed of light as a constant, determine distances. He called his new instrument a Geodimeter. The name Geodimeter was made up from the letters Geo, Di, and Meter, which means geodetic distance measurement.

Note: Discussion open until September 1, 1958. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. Paper 1617 is part of the copyrighted Journal of the Surveying and Mapping Division, Proceedings of the American Society of Civil Engineers, Vol. 84, No. SU 1, April, 1958.

a. Paper presented at Annual Meeting, ASCE, October 14, 1957, New York, N. Y.

1. Supervisory Mathematician, U. S. Coast and Geodetic Survey, Washington, D. C.

The Geodimeter

Dr. Bergstrand arranged with the AGA Company of Stockholm, Sweden, for the commercial production of his Geodimeter and Models 1 and 2 were soon available. Model 1 is a two frequency instrument and Model 2 is a three-frequency instrument.

Fig. 1 is a schematic diagram of the operation of the Geodimeter. A beam of light is transmitted through crossed polaroids and a Kerr cell, which modulates the intensity of the light beam at a frequency of 10 Mc/sec. This modulated light beam travels to a distant mirror, or bank of retrodirective prisms, which reflects it back to a photocell where it is detected. The returning light beam is instantaneously compared in phase with that of the light just leaving the Geodimeter by a suitable phase detector. The phase detector indicator is called a null detector and indicates zero when the outgoing and incoming light beams are in phase. A suitable variable electrical delay is inserted between the outgoing light modulator and the phase detector so the phases can be made to agree.

Fig. 2 illustrates a Geodimeter and mirror system set up to measure a line. It is assumed that the length of the line is a multiple of 7.5 meters which is a quarter wavelength of the accurately controlled 10 megacycle per second modulating frequency. Hence the null indicator reads zero. If the Geodimeter is now moved away from the mirror a short distance the null indicator is now to the right or left of zero. Instead of moving the Geodimeter an electrical delay on the instrument can be changed to zero the null indicator. In effect the instrument has been moved electrically. A line to be measured will always contain an integral number of 7.5 meter units plus a fractional part of 7.5 meters. The Geodimeter is provided with a built-in variable light path which is calibrated in units and submultiples of meters. The light can be diverted from the distant mirror, and the electrical delay, which is indicating the fraction of the 7.5 meter interval can be resolved into a distance by the transmission of the light beam through the built-in calibrated light path. This gives the submultiple of 7.5 meters, but we still do not know the integral number of 7.5 meter units contained in the line. The Model 2 instrument is provided with three modulating frequencies which, in effect, provides units in space of three different lengths, thus it is not necessary to know the approximate length of the line being measured as closely as with a two-frequency instrument, such as the Model 1.

Fig. 3 illustrates an instrument with two modulating frequencies in which 100 units in space of one frequency is equal to 101 units of the other frequency. From that relationship, and knowing if there is an odd or even number of these units; it is possible to find to whole number of units when measuring, in terms of each different unit. It is necessary to know the length of the line being measured with a two-frequency geodimeter within ± 750 meters, but the three frequency Model 2 will resolve ± 1500 meters.

The Geodimeter is composed physically of two sections, the measuring unit weighing 108 pounds and the optical unit weighing 113 pounds. Each unit is packed in its own shock absorbing wooden case. The power for the Geodimeter should be 115 or 230 volts, 60 cycle alternating current at 140 watts. The U. S. Coast and Geodetic Survey uses a 230 volt 60 cycle, 300-watt gasoline generator, weighing about 30 pounds. For mounting the instrument, a sturdy metal tripod with a revolving top plate should be provided. The Geodimeter should be protected during observations with a suitable tent, to keep

dew and moisture off the optics and out of the electrical components. The mirror used with the Geodimeter at present is composed of 32 retrodirective prisms mounted in a light metal frame and weighs about 30 pounds. The mirror system is mounted on a surveying type tripod which has a head adapted to the mirror mounting. The retrodirective prism system has been found to be more satisfactory than the plane mirror. It will direct the light back even when the pointing is off several degrees. In addition to the Geodimeter and mirror system, each end of the line being measured should be provided with a thermometer, barometer (or altimeter), psychrometer, and two-way radio. A variac to adjust the generator voltage for the geodimeter is also desirable.

It has been found from experience that a total of 12 observations, taken on two different nights, is sufficient to give a length suitable for first-order triangulation. Temperature and barometric pressure are taken on each end of the line, before and after each set of observations. Relative humidity is measured at the beginning and end of a night's work. Mean temperatures and pressures over the line, both before and after a complete observation, along with relative humidity, are used to compute the refraction correction.

The procedure for a night's work follows a pattern which is relatively simple. The geodimeter is mounted on its tripod with the back edge over the station mark, and all optics and associated parts are put in place. The gasoline generator is started and The Geodimeter cord plugged into it. While the Geodimeter electronic circuits are warming up, the observer inserts the sighting telescope in the optical line of sight of the instrument and centers on the light being shown at the mirror end. The observer then removes the telescope and swings the lamp assembly into place and instructs the mirror tender, by radio, to replace his signal lamp with the mirror system. After the Geodimeter electronic circuits have been turned on for an hour, and it is sufficiently dark, observations can begin. The Geodimeter is then further lined up optically for best light return. Minor adjustments are made in the optical system for best strength of light return as indicated on the light indicator meter. Temperature, pressure, and humidity readings are taken at both ends of the line and the light intensity is adjusted for proper operation.

A set of 6 measurements of one night's work usually can be made in one and one-half hours. Fig. 4 is a typical sheet of Geodimeter observations for two measurements of a line, one on each of two modulating frequencies. Two mean light conductor readings plus a mean mirror reading give sufficient data to obtain an electrical delay reading which can be transformed into a distance equivalent. Fig. 5 is a sample computation of length for a Geodimeter line. The results from observations taken with three frequencies are listed and the different parts making up the final lengths are labeled.

The U. S. Coast and Geodetic Survey has measured 56 lines with the Geodimeter. Four of these lines were over first-order taped bases. The agreements in lengths were 1 part in 158,000, one part in 177,000, 1 part in 535,000, and 1 part in 508,000. Many of the Geodimeter lines have been used directly in triangulation adjustments as first-order bases, with good agreement against adjusted triangulation. In three instances a Geodimeter length was measured in a quadrilateral containing a taped base. These gave agreements ranging from 1 part in 143,000 to 1 part in 1,500,000 when compared with the lengths derived from the taped base.

The relative humidity is generally high at night when the Geodimeter is used. A range of relative humidity up to 100 per cent will introduce an error

of only 1 part in one million in the length of the line being measured. An error of temperature of 1° C, or altimeter error of 100 feet, will each introduce an error of one part in a million in the length of the line being measured.

Like any precise instrument, the Geodimeter should be transported with care. Shock mountings should be provided in the transporting vehicle. Rough handling will cause misalignment of the optics and cause breaks in the electrical system.

The Tellurometer

The Telecommunications Research Laboratory of the South African Council for Scientific and Industrial Research recently developed an instrument for the measurement of geodetic distances. It has been named the Tellurometer, the development of which was under the direction of Mr. T. L. Wadley of that laboratory.

The Tellurometer utilizes radio microwaves to transmit measuring information between a master and a remote station. It is a phase-comparison system similar to other phase-comparison systems used for distance and range measurement. It operates on the principle of the instantaneous sampling of the phase of independent oscillators as the master and remote stations. The phase relationship of these oscillators, at the remote station, is relayed back to the master station and compared on a phase-indicating device at the master station. The indicator used to compare the phases is a cathode ray tube, similar to a small television picture tube, which displays a circular pattern. The circular pattern is broken at one place on the circle, which is the indication of the phase comparison at the remote station, between the master and remote stations. The cathode ray tube has a circular scale of 100 units around the circumference, from which readings can be made to the nearest even units or the nearest half unit by estimation.

The Tellurometer uses radio microwaves instead of light waves as the transmitting medium and like the Geodimeter uses 10 Mc/sec. modulation to produce the phase-comparison information. In the Tellurometer the cathode ray tube with its scale replaces the function of the null indicator of the Geodimeter.

If the master station is moved away from or towards the remote station it is found that the break in the circular sweep on the cathode ray tube revolves one complete revolution for every 50 feet movement of the master station. Thus any line to be measured will contain an integral number of 50 foot units plus a fractional part of 50 feet. Crystal modulating frequencies noted as A+ and A- are used to determine fractional parts of 50 feet and crystals denoted by the letters B, C, and D are used to measure fractional parts of 50,000, 5,000 and 500 feet respectively. Thus the length of the line being measured must be known only to the nearest 10 miles.

The master and remote stations are similar but not identical and are of equal size and weight. The complete equipment for a master or remote station consists of the Tellurometer instrument, tripod, power pack, and battery. Each master and remote unit has an integrated radio telephone system which uses the measuring frequencies for voice transmission. However simultaneous communication and measuring is not possible, but is controlled alternately by means of a switch on the instrument. In addition, it is necessary to have a barometer and psychrometer if precise results are desired. The minimum

equipment for measuring is a master and remote unit. Several remote units can be used with one master station for the measurement of several lines. Each unit is contained in a steel carrying case and is sufficiently shock mounted for transportation by truck or car. The total weight of a unit is approximately 85 pounds. This is composed of the Tellurometer instrument which weighs 24 lbs., carrying case 14 lbs., power pack 10 lbs., tripod 9 lbs., and battery 28 lbs. The battery recommended for the instrument is a 6-volt 40 ampere-hour storage battery. Each Tellurometer unit uses approximately 7 amperes at 6.3 volts. Straps are provided for the carrying case and power pack so the instrument can be packed to a station not accessible by truck or car.

The total time for the measurement of a line is approximately one-half hour, which includes setting up the equipment, measuring, and packing the equipment for transportation. Any number of observations can be used for a length measurement, but two coarse sets and a set of 12 fine readings are usually sufficient for geodetic accuracy. The Tellurometer requires a line-of-sight, but visibility is not necessary. The instrument can be used in the day or at night, but best results are obtained on a dry, clear day with a wind blowing along the line being measured. In order to obtain good results, it is necessary to make careful measurements of humidity, temperature, and pressure at both ends of the line, both at the beginning and end of a set of observations.

It is a short and simple procedure to take a set of observations with the Tellurometer. The Tellurometer, at both the master and remote stations, is mounted on the tripod and centered over the mark. The antenna reflector is placed in the proper place and fastened to the unit. The power supply unit is connected to the Tellurometer and to the battery, and the low voltage (L T) switch is turned on. After one or two minutes the high voltage switch (H T) is turned on and the instrument is ready to operate.

A typical record sheet for the measurement of a line is shown in Fig. 6. The A crystal temperature reading is taken at the master station and recorded. Wet and dry bulb temperatures and barometric readings are taken at both stations and recorded in the appropriate places on the data sheet. A coarse reading is now taken for the length of the line. An A+ reading is made and recorded and then A-, B, C, and D readings are made and recorded. The remote station operator gives the appropriate A+, A-, B, C, and D signals upon instruction of the master station operator. The coarse reading is obtained by subtracting the B, C, and D values respectively from the A+ reading, and using only the first digit on the A minus B, A minus C, and A minus D readings. Thus, in the example, the first digit obtained is a 1 by knowledge that the line is over ten and less than twenty miles. The second digit is the 1 from A minus B then the 3 from A minus C, 0 from A minus D. The 77 is the mean of the A+ and A-readings. The time of transit of the signals in millimicroseconds thus is 113077. This is a rough determination of the time for the radio waves to travel from the master to remote station and return. A set of 12 fine readings is now taken to get a more accurate distance for the line. The remote operator furnishes A+, A-, A+ reversed, and A- reversed signals as requested by the master station. Each set of four is averaged to get an A fine reading. Forty-eight individual readings are resolved into 12 fine readings which are averaged to get a final "A" reading value. A final coarse reading is now taken and crystal temperature at the master station is read and recorded. Wet and dry bulb temperatures and barometric pressure are taken at each station and recorded.

The mean "A" crystal temperature is computed and compared with the standard operating temperature for the crystal. If the change is significant, the frequency of the crystal is corrected for this change in temperature.

United States Weather Bureau tables are used for computing the dew point and vapor pressure. They use wet and dry bulb temperatures, and barometric pressure as arguments. Mean vapor pressure is computed and listed on the refractive index computation sheet. The mean dry bulb temperature and mean barometric pressure are also recorded. A nomogram or formula can be used to compute the refraction correction. The nomogram is easy to use and in most cases is sufficiently accurate. (See Fig. 8) Using a straightedge, the vapor pressure and temperature are used as arguments to get the equivalent pressure E. This equivalent pressure is added to the mean barometric pressure to get a total effective pressure (P + E). Using total effective pressure and temperature as arguments, the nomogram gives the refractive index. The length of the line measured is one-half the speed of radio waves multiplied by the transit time determined from the Tellurometer observations, divided by the refractive index. See Fig. 7.

The Tellurometer manufacturer on the basis of preliminary tests of the instrument stated that the accuracy to be expected would be 2 inches plus 3 parts per million of the length of the line being measured. Tests were made with the Tellurometer in England in the spring of 1957 with excellent results. The manufacturers of the instrument measured 10 adjusted triangulation lines, of lengths between 11,000 and 73,000 meters, and obtained checks ranging from 1 part in 230,000 to 1 part in 1,630.000. The Coast and Geodetic Survey Tellurometer was tested on only one short line before being sent to Alaska to do traverse, but checked within the manufacturers stated limit on that line. The U. S. Coast and Geodetic Survey hopes to measure sufficient lines in the near future to give an independent evaluation of the accuracy and performance of the Tellurometer. The Geodimeter and the Tellurometer can be used for many kinds of distance measurement. Examples are base lines, traverse, trilateration, or any surveying application where a precise distance is required. The Geodimeter has been operated from a tower, but it is better to use a ground station for the instrument and the mirror on the tower. The Geodimeter must be used at night and visibility is necessary over the line being measured. The Geodimeter is not easily carried for any distance due to the weight, but is much less costly and quicker to use for base line measurement than conventional taping. The Geodimeter has always given an accuracy sufficient for first-order bases when it is operating properly. The Tellurometer can be back-packed to a station, or it can be mounted on a tower. Although it can be used at night, it is preferable to make the observations during the day at a time when atmospheric conditions are settled. The Tellurometer requires a line of sight but not visibility. It can be operated in fog, haze, or light rain. Criteria for the selection of station sites for lines to be measured are about the same for both instruments.

The Geodimeter can be used over any terrain in which the light beam will travel to the mirror and return. The limiting factor in precise measurement with the Geodimeter is the accuracy with which the refraction can be determined. For that reason it is preferable to have good clearance over terrain or water, since the temperature and pressure over the line will be more uniform, hence the refraction can be more precisely determined. The Tellurometer should have good clearance along the line not only to get a better refraction determination, but to prevent reflections of the radio waves from

objects which may be near the line of sight. Experience has shown that when a line is grazing the ground at the middle it is sometimes impossible to get a signal through to make a measurement. In these cases the line can be successfully determined by measuring it in two parts. Tellurometer lines have been measured over large bodies of water with good results. It is preferable when measuring over water to have the line near the water. Lines measured over water from elevated stations at each end sometimes are difficult to measure due to the reflections of the radio waves off the water surface. Any type of terrain which will tend to break up reflections of the radio waves, such as trees, brush, and tall vegetation, is desirable when measuring with the Tellurometer.

The ability to measure distances directly by electronic and electronic-optical methods will furnish economical means for conducting traverse surveys, not only in relatively flat areas, but also over rugged terrain where previously such surveys would have been impossible.

The Geodimeter and the Tellurometer are definitely valuable surveying instruments which will play a tremendous part in future geodetic and engineering surveys.

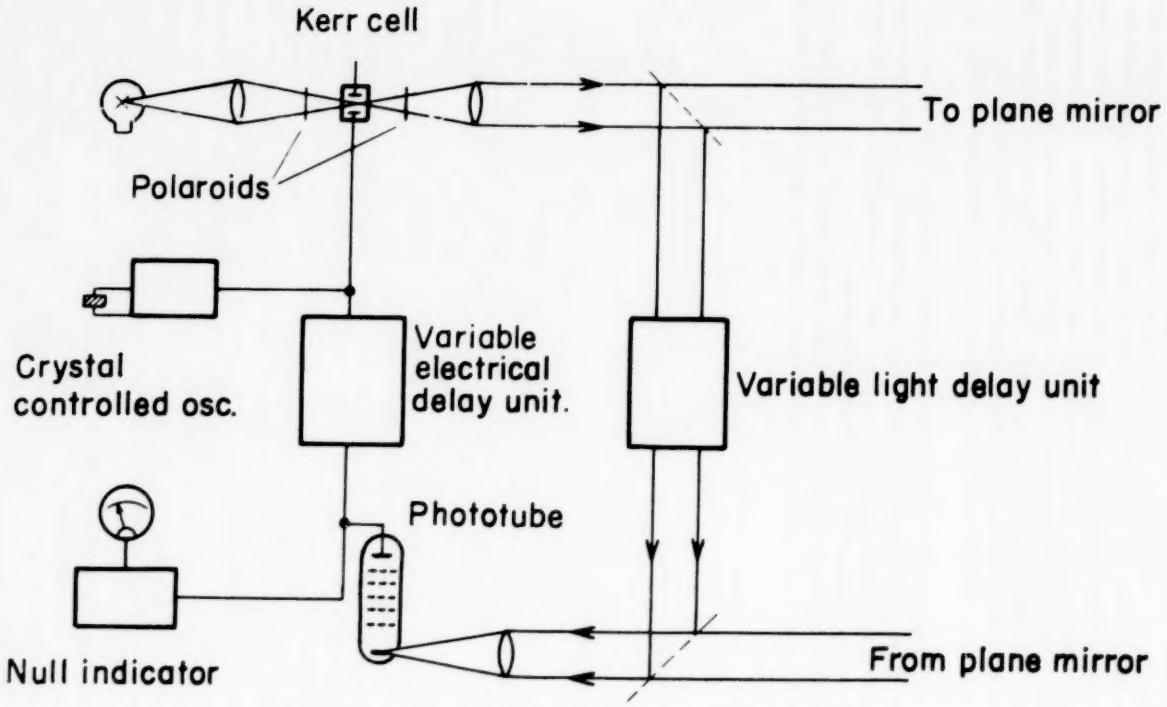


Fig. 1 Schematic diagram of the geodimeter.

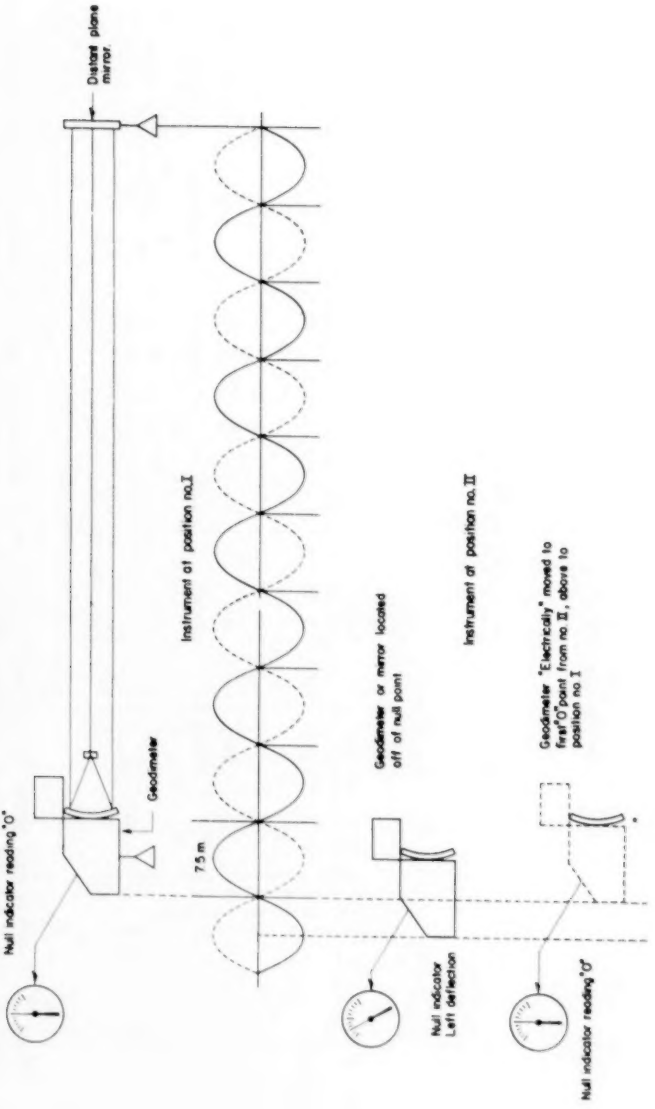

Distant plane mirror
Instrument at position no.I
Geodimeter
7.5 m
Null indicator reading "0"
Geodimeter or mirror located off of null point
Null indicator Left deflection
Instrument at position no.II
Geodimeter "Electrically" moved to first "0" point from no II, above to position no I
Null indicator reading "0"

Fig. 2 Null indicator reading effects with geodimeter (or reflex) on and off null points.

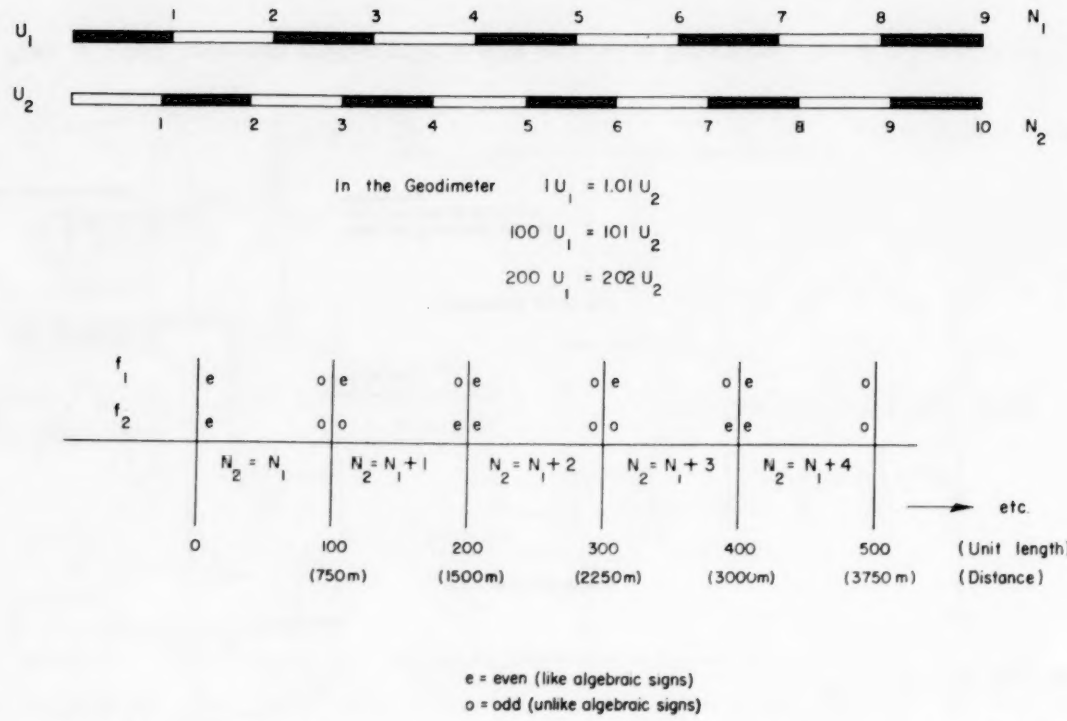

U1
1 2 3 4 5 6 7 8 9 N1
U2
1 2 3 4 5 6 7 8 9 10 N2
In the Geodimeter 1 U1 = 1.01 U2
100 U1 = 101 U2
200 U1 = 202 U2
f1
f2
e o e o e o e o e o
e o o e e o o e e o
N2 = N1
N2 = N1 + 1
N2 = N1 + 2
N2 = N1 + 3
N2 = N1 + 4
etc.
0 100 200 300 400 500 (Unit length)
(750m) (1500m) (2250m) (3000m) (3750m) (Distance)
e = even (like algebraic signs)
o = odd (unlike algebraic signs)

Fig. 3 A scale and table to illustrate how two frequencies and a knowledge of the combination of signs may be used to resolve the ambiguity of 750 meters in computing distances.

Form 23 (8-6-54) Figure 4

U.S. DEPARTMENT OF COMMERCE
COAST AND GEODETIC SURVEY

GEODIMETER OBSERVATIONS

State MISSISSIPPI Locality GREENWOOD Date 2/21/57
Station CASCILLA to Station DUNCANS Geod. No. 114
Observer G.B. LESLEY Height Geodimeter 1 Meters
Mirror Tender J.D. RIGNEY Height Mirror 30 Meters

Time Heater on 17:10 Chief of Party C.A. SHELTON
Time Plate on 17:14 Recorder C.A. SHELTON
Lamp Voltage 3.6 Focus, Transmitting 10.3
Heater Voltage 6.3 Focus, Receiving 9.3

Frequency	El Dl	No. 1 Lt. Cond.	Mirror	No. 2 Lt. Cond.	Frequency	El Dl	No. 1 Lt. Cond.	Mirror	No. 2 Lt. Cond.
1		9-40		9-20	2		7-80		7-70
Phase	8				Phase	6			
1		+ 44.4	- 40.0	+ 29.5	1		+ 53.9	- 54.8	+ 43.2
2		- 36.4	+ 24.1	- 22.2	2		- 47.2	+ 37.7	- 39.2
3		+ 36.2	- 23.8	+ 21.8	3		+ 46.3	- 37.8	+ 39.0
4		- 44.9	+ 40.2	- 30.6	4		- 53.8	+ 53.7	- 44.0
Sum		161.9	128.1	104.1	Sum		201.2	184.0	165.4
Mean		40.48	32.02	26.02	Mean		50.30	46.00	41.35

	Mirror	Geodimeter	Mirror	Geodimeter	Mirror	Geodimeter
Time		18:20		18:32		18:50
Temp.	11.0	10.0	10.9	10.0	10.8	10.0
ALT. FEET Press.	1330	1272	1328	1272	1325	1272
R.Hu.	52 W 65 D	39%				

Eccentricity, Geodimeter +0.0280 Meters Mirror -0.1930 Meters

Reduction of Electrical Length to Physical Length:

LC. Light Conductor, course. lc. Light Conductor, fine. m. Mirror, fine

F1

Correct lc LC_1 40 $- \frac{lc_1}{lc_1} \frac{40.48}{40.48} \cdot m \frac{32.02}{lc_2 26.02}$ LC_1 40 $- LC_2$ 20 = 28.30

F2

Correct lc LC_1 80 $- \frac{lc_1}{lc_1} \frac{50.30}{50.30} \cdot m \frac{46.00}{lc_2 41.35}$ LC_1 80 $- LC_2$ 70 = 75.20

Weather: Partly Cloudy, Clear, Haze, Smoke, Sharp
Thermometer: 2 Meters above ground at Geodimeter
30 Meters above ground at Mirror
Wind: Calm, Light, Moderate, Hard
Remarks:

Arrow in wind direction

Coast-DC 61163

Figure 5 GEODIMETER OBSERVATIONS

Station: Cascilla to Duncans Date 2/21/57
State: Mississippi Locality: Greenwood

constants: Frequency 1 = 9,999,996 cycles
Frequency 2 = 10,049,902 cycles
Frequency 3 = 10,299,847 cycles
Speed of light = 299,793.1 km/sec.

$$\frac{299{,}793.1}{4\,(\,9{,}999{,}996\,)(\,1.0002894\,)} = 7.49266212 \text{ meters (Frequency 1)}$$

$$\frac{299{,}793.1}{4\,(10{,}049{,}902\,)\,(1.0002895\,)} = 7.45545417 \text{ meters (Frequency 2)}$$

$$\frac{299{,}793.1}{4\,(10{,}299{,}847\,)\,(1.0002896\,)} = 7.27453294 \text{ meters (Frequency 3)}$$

1573 x 7.49266212 =	11785.9575	meters
8 x 0.7995 =	6.3960	"
partial light conductor	0.2830	"
Zero correction	1.1288	"
Eccentricity	-0.1650	"
Focus correction	-0.0109	"
Mirror correction	-0.0088	"
Length (Freq. 1)	11793.5806	meters

1581 x 7.45545417 =	11787.0730	meters
6 x 0.7995	4.7970	"
partial light conductor	0.7520	"
Zero correction	1.1288	"
Eccentricity	-0.1650	"
Focus correction	-0.0109	"
Mirror correction	-0.0088	"
Length (Freq. 2)	11793.5661	meters

1621 x 7.27453294 =	11792.0179	meters
0 x 0.7995	0.0000	
partial light conductor	0.6191	"
Zero correction	1.1288	"
Eccentricity	-0.1650	"
Focus correction	-0.0109	"
Mirror correction	-0.0088	"
Length (Freq. 3)	11793.5811	meters

Figure 6

TELLUROMETER FIELD SHEET

MASTER STATION: KINGS BTY. PE INST. No. MA 2 OPERATOR R.G. Stewart

REMOTE STATION: KLIPFONTEIN INST. No. RA 2 OPERATOR R. Coulter

DATE MARCH 5, 1957 CONDITIONS Blustery, Dry, Wind 30 Knots across line

TIME 16:30 - 17:05

Note: Add 100 as A- reading is in effect A+ reading taken anti-clockwise

COARSE READINGS

Initial				Final			
A+74	A+74	A+74	A+74	A+76	A+76	A+76	A+76
B 63	C 44	D 66	A-22	B 65	C 45	D 67	A-22
11	30	09	$\frac{152}{2}=76$	11	31	09	$\frac{154}{2}=77$

Approx. Distance 11 MILES Coarse Figure = 11 30 77

MET. READINGS

	Crystal Temp.	Dry Bulb °F	Wet Bulb °F	Dep. °F	Dew Point	Vap. Pres.	Alt. Rdg. ft.	Baro. Correct ins.	Cor'ted Baro. ins.	Alt. Serial No.
Master Initial	65	62	58	4	55	0.432		NIL	28.68	Master
Final	60	62	57	5	54	0.417		"	28.66	
Remote Initial		72	62	10	56	0.448		"	29.96	
Final		70	62	8	57	0.465		"	29.96	Remote
Sum	125	266				1.762			117.26	
Mean	62.5	66.5				0.4405			29.32	

FINE READINGS

Remote Setting	A+ A-	A+R A-R	Fine Reading	Remote Setting	A+ A-	A+R A-R	Fine Reading
	74.5 22.5 52	26.5 75.5 51	75.75		76 22.5 53.5	27 75 52	76.38
	76 23 53	26.5 75.5 51	76.00		76 23 53	27 75.5 51.5	76.12
	76 23 53	26.5 75 51.5	76.12		76.5 23 53.5	27.5 74.5 53	76.62
	76 23 53	27 75 52	76.25		76.5 22 54.5	26 74.5 51.5	76.50
	76.5 23 53.5	27.5 74 53.5	76.75		76 21 55	27.5 74 53.5	77.12
	76 22.5 53.5	27 76 51	76.12		76.5 21 55.5	27.5 75 52.5	77.00
						Mean	76.39

Page _____

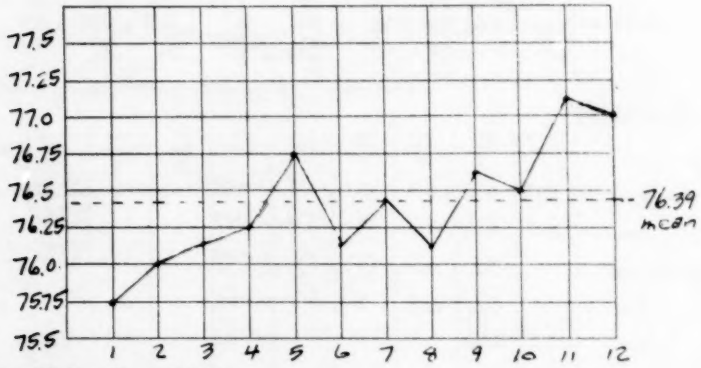
Figure 7 TELLUROMETER REDUCTION SHEET

Master Station KINGS GTY PEG. Inst. No. MA 2

Remote Station KLIPFONTEIN Inst. No. RA 2

Coarse Figure 113077.00 m μ s

Fine Figure 113076.39 m μ s (Mean uncorrected value)



Refractive Index Computation

Mean Barometer 29.32 in. Mean Vapor Pressure 0.4405 in.

Equiv. Vapor Pres. + 7.2 in. n = 1.000 329

Total Eff. Pressure 36.52 in.

Crystal Temperature Correction

Uncorrected Transit Time 113076.39 m μ s

Crystal Correction 1 P.P.M. no corr m μ s

Corrected Transit Time 113076.39 m μ s $T_c = \frac{\text{Corr Trans Time}}{n} = \frac{113039.31}{\text{m } \mu \text{ s}}$

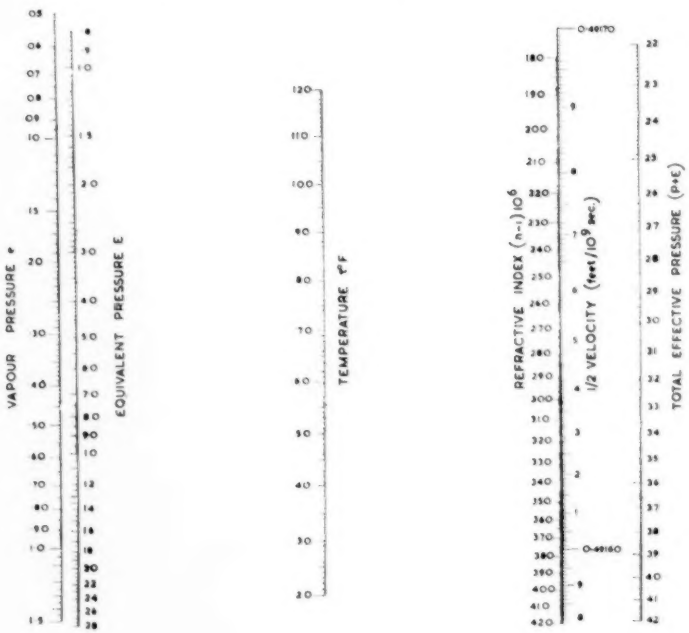
Velocity Factor = 0.1498965 meters/ m μ s

s = Final slope distance = (0.1498965)(T_c) = 16944.1969 meters

Note: When using nomograph lay ruler across mean temperature and mean vapor pressure scales and read off equivalent pressure. Add equivalent pressure to mean barometer to obtain total effective pressure. Lay ruler across mean temperature and total effective pressure and read off last three figures of refractive index (n).

FIGURE 8

REFRACTIVE INDEX OF RADIO WAVES



Formula: $(n-1)\,10^6 = \frac{4730}{459{\cdot}9+t}(P+E)$

where

$E = \frac{8938}{459{\cdot}9+t}e$

t = Air temperature in °F

P = Barometric pressure in inches of mercury

e = Vapour pressure in inches of mercury

v = Velocity in vacuo = 299 792·0 Km/sec.
= 0·983 571 3×10⁹ Feet/sec

½v = ½ Velocity in vacuo = 0·491 785 6×10⁹ Feet/sec

M351

PROCEEDINGS PAPERS

The technical papers published in the past year are identified by number below. Technical-division sponsorship is indicated by an abbreviation at the end of each Paper Number, the symbols referring to: Air Transport (AT), City Planning (CP), Construction (CO), Engineering Mechanics (EM), Highway (HW), Hydraulics (HY), Irrigation and Drainage (IR), Pipeline (PL), Power (PO), Sanitary Engineering (SA), Soil Mechanics and Foundations (SM), Structural (ST), Surveying and Mapping (SU), and Waterways and Harbors (WW), divisions. Papers sponsored by the Board of Direction are identified by the symbols (BD). For titles and order coupons, refer to the appropriate issue of "Civil Engineering." Beginning with Volume 82 (January 1956) papers were published in Journals of the various Technical Divisions. To locate papers in the Journals, the symbols after the paper numbers are followed by a numeral designating the issue of a particular Journal in which the paper appeared. For example, Paper 1449 is identified as 1449 (HY 6) which indicates that the paper is contained in the sixth issue of the Journal of the Hydraulics Division during 1957.

VOLUME 83 (1957)

APRIL: 1196(EM2), 1197(HY2), 1198(HY2), 1199(HY2), 1200(HY2), 1201(HY2), 1202(HY2), 1203(SA2), 1204 (SM2), 1205(SM2), 1206(SM2), 1207(SM2), 1208(WW1), 1209(WW1), 1210(WW1), 1211(WW1), 1212(EM2), 1213(EM2), 1214(EM2), 1215(PO2), 1216(PO2), 1217(PO2), 1218(SA2), 1219(SA2), 1220(SA2), 1221(SA2), 1222(SA2), 1223(SA2), 1224(SA2), 1225(PO)[c], 1226(WW1)[c], 1227(SA2)[c], 1228(SM2)[c], 1229(EM2)[c], 1230 (HY2)[c].

MAY: 1231(ST3), 1232(ST3), 1233(ST3), 1234(ST3), 1235(IR1), 1236(IR1), 1237(WW2), 1238(WW2), 1239(WW2), 1240(WW2), 1241(WW2), 1242(WW2), 1243(WW2), 1244(HW2), 1245(HW2), 1246(HW2), 1247(HW2), 1248 (WW2), 1249(HW2), 1250(HW2), 1251(WW2), 1252(WW2), 1253(IR1), 1254(ST3), 1255(ST3), 1256(HW2), 1257 (IR1)[c], 1258(HW2)[c], 1259(ST3)[c].

JUNE: 1260(HY3), 1261(HY3), 1262(HY3), 1263(HY3), 1264(HY3), 1265(HY3), 1266(HY3), 1267(PO3), 1268 (PO3), 1269(SA3), 1270(SA3), 1271(SA3), 1272(SA3), 1273(SA3), 1274(SA3), 1275(SA3), 1276(SA3), 1277 (HY3), 1278(HY3), 1279(PL2), 1280(PL2), 1281(PL2), 1282(SA3), 1283(HY3)[c], 1284(PO3), 1285(PO3), 1286 (PO3), 1287(PO3)[c], 1288(SA3)[c].

JULY: 1289(SM3), 1290(EM3), 1291(EM3), 1292(EM3), 1293(EM3), 1294(HW3), 1295(HW3), 1296(HW3), 1297 (HW3), 1298(HW3), 1299(SM3), 1300(SM3), 1301(SM3), 1302(ST4), 1303(ST4), 1304(ST4), 1305(SU1), 1306 (SU1), 1307(SU1), 1308(ST4), 1309(SM3), 1310(SU1)[c], 1311(EM3)[c], 1312(ST4), 1313(ST4), 1314(ST4), 1315 (ST4), 1316(ST4), 1317(ST4), 1318(ST4), 1319(SM3)[c], 1320(ST4), 1321(ST4), 1322(EM3), 1323(AT1), 1324 (AT1), 1325(AT1), 1326(AT1), 1327(AT1), 1328(AT1)[c], 1329(ST4)[c].

AUGUST: 1330(HY4), 1331(HY4), 1332(HY4), 1333(SA4), 1334(SA4), 1335(SA4), 1336(SA4), 1337(SA4), 1338 (SA4), 1339(CO1), 1340(CO1), 1341(CO1), 1342(CO1), 1343(CO1), 1344(PO4), 1345(HY4), 1346(PO4)[c], 1347 (BD1), 1348(HY4)[c], 1349(SA4)[c] 1350(PO4), 1351(PO4).

SEPTEMBER: 1352(IR2), 1353(ST5), 1354(ST5), 1355(ST5), 1356(ST5), 1357(ST5), 1358(ST5), 1359(IR2), 1360 (IR2), 1361(ST5), 1362(IR2), 1363(IR2), 1364(IR2), 1365(WW3), 1366(WW3), 1367(WW3), 1368(WW3), 1369 (WW3), 1370(WW3), 1371(HW4), 1372(HW4), 1373(HW4), 1374(HW4), 1375(PL3), 1376(PL3), 1377(IR2)[c], 1378 (HW4)[c], 1379(IR2), 1380(HW4), 1381(WW3)[c], 1382(ST5)[c], 1383(PL3)[c], 1384(IR2), 1385(HW4), 1386(HW4).

OCTOBER: 1387(CP2), 1388(CP2), 1389(EM4), 1390(EM4),1391(HY5), 1392(HY5), 1393(HY5), 1394(HY5), 1395 (HY5), 1396(PO5), 1397(PO5), 1398(PO5), 1399(EM4), 1400(SA5), 1401(HY5), 1402(HY5), 1403(HY5), 1404 (HY5), 1405(HY5), 1406(HY5), 1407(SA5), 1408(SA5), 1409(SA5), 1410(SA5), 1411(SA5), 1412(EM4), 1413 (EM4), 1414(PO5), 1415(EM4)[c], 1416(PO5)[c], 1417(HY5)[c], 1418(EM4), 1419(PO5), 1420(PO5), 1421(PO5), 1422(SA5)[c], 1423(SA5), 1424(EM4), 1425(CP2).

NOVEMBER: 1426(SM4), 1427(SM4), 1428(SM4), 1429(SM4), 1430(SM4)[c], 1431(ST6), 1432(ST6), 1433(ST6), 1434(ST6), 1435(ST6), 1436(ST6), 1437(ST6), 1438(SM4), 1439(SM4), 1440(ST6), 1441(ST6), 1442(ST6)[c], 1443(SU2), 1444(SU2), 1445(SU2), 1446(SU2), 1447(SU2), 1448(SU2)[c].

DECEMBER: 1449(HY6), 1450(HY6), 1451(HY6), 1452(HY6), 1453(HY6), 1454(HY6), 1455(HY6), 1456(HY6)[c], 1457(PO6), 1458(PO6), 1459(PO6), 1460(PO6)[c], 1461(SA6), 1462(SA6), 1463(SA6), 1464(SA6), 1465(SA6), 1466(SA6)[c], 1467(AT2), 1468(AT2), 1469(AT2), 1470(AT2), 1471(AT2), 1472(AT2), 1473(AT2), 1474(AT2), 1475(AT2), 1476(AT2), 1477(AT2), 1478(AT2), 1479(AT2), 1480(AT2), 1481(AT2), 1482(AT2), 1483(AT2), 1484(AT2), 1485(AT2)[c], 1486(BD2), 1487(BD2), 1488(PO6), 1489(PO6), 1490(BD2), 1491(BD2), 1492(HY6), 1493(BD2).

VOLUME 84 (1958)

JANUARY: 1494(EM1), 1495(EM1), 1496(EM1), 1497(IR1), 1498(IR1), 1499(IR1), 1500(IR1), 1501(IR1), 1502 (IR1), 1503(IR1), 1504(IR1), 1505(IR1), 1506(IR1), 1507(IR1), 1508(ST1), 1509(ST1), 1510(ST1), 1511(ST1), 1512(ST1), 1513(WW1), 1514(WW1), 1515(WW1), 1516(WW1), 1517(WW1), 1518(WW1), 1519(ST1), 1520 (EM1)[c], 1521(IR1)[c], 1522(ST1)[c], 1523(WW1)[c], 1524(HW1), 1525(HW1), 1526(HW1)[c], 1527(HW1).

FEBRUARY: 1528(HY1), 1529(PO1), 1530(HY1), 1531(HY1), 1532(HY1), 1533(SA1), 1534(SA1), 1535(SM1), 1536(SM1), 1537(SM1), 1538(PO1)[c], 1539(SA1), 1540(SA1), 1541(SA1), 1542(SA1), 1543(SA1), 1544(SM1), 1545(SM1), 1546(SM1), 1547(SM1), 1548(SM1), 1549(SM1), 1550(SM1), 1551(SM1), 1552(SM1), 1553(PO1), 1554(PO1), 1555(PO1), 1556(PO1), 1557(SA1)[c], 1558(HY1)[c], 1559(SM1)[c].

MARCH: 1560(ST2), 1561(ST2), 1562(ST2), 1563(ST2), 1564(ST2), 1565(ST2), 1566(ST2), 1567(ST2), 1568 (WW2), 1569(WW2), 1570(WW2), 1571(WW2), 1572(WW2), 1573(WW2), 1574(PL1), 1575(PL1), 1576(ST2)[c], 1577(PL1), 1578(PL1)[c], 1579(WW2)[c].

APRIL: 1580(EM2), 1581(EM2), 1582(HY2), 1583(HY2), 1584(HY2), 1585(HY2), 1586(HY2), 1587(HY2), 1588 (HY2), 1589(IR2), 1590(IR2), 1591(IR2), 1592(SA2), 1593(SU1), 1594(SU1), 1595(SU1), 1596(EM2), 1597(PO2), 1598(PO2), 1599(PO2), 1600(PO2), 1601(PO2), 1602(PO2), 1603(HY2), 1604(EM2), 1605(SU1)[c],1606(SA2), 1607(SA2), 1608(SA2), 1609(SA2), 1610(SA2), 1611(SA2), 1612(SA2), 1613(SA2), 1614(SA2)[c], 1615(IR2)[c], 1616 (HY2)[c], 1617(SU1), 1618(PO2)[c], 1619(EM2)[c], 1620(CP1).

c. Discussion of several papers, grouped by divisions.

AMERICAN SOCIETY OF CIVIL ENGINEERS

OFFICERS FOR 1958

PRESIDENT

LOUIS R. HOWSON

VICE-PRESIDENTS

Term expires October, 1958:
FRANCIS S. FRIEL
NORMAN R. MOORE

Term expires October, 1959:
WALDO G. BOWMAN
SAMUEL B. MORRIS

DIRECTORS

Term expires October, 1958:
JOHN P. RILEY
CAREY H. BROWN
MASON C. PRICHARD
ROBERT H. SHERLOCK
R. ROBINSON ROWE
LOUIS E. RYDELL
CLARENCE L. ECKEL

Term expires October, 1959:
CLINTON D. HANOVER, Jr.
E. LELAND DURKEE
HOWARD F. PECKWORTH
FINLEY B. LAVERTY
WILLIAM J. HEDLEY
RANDLE B. ALEXANDER

Term expires October, 1960:
PHILIP C. RUTLEDGE
WESTON S. EVANS
TILTON E. SHELBURNE
CRAIG P. HAZELET
DONALD H. MATTERN
JOHN E. RINNE

PAST PRESIDENTS

Members of the Board

ENOCH R. NEEDLES

MASON G. LOCKWOOD

EXECUTIVE SECRETARY
WILLIAM H. WISELY

TREASURER
CHARLES E. TROUT

ASSISTANT SECRETARY
E. LAWRENCE CHANDLER

ASSISTANT TREASURER
CARLTON S. PROCTOR

PROCEEDINGS OF THE SOCIETY

HAROLD T. LARSEN
Manager of Technical Publications

PAUL A. PARISI
Editor of Technical Publications

FRANCIS J. SCHNELLER, JR.
Assistant Editor of Technical Publications

COMMITTEE ON PUBLICATIONS

HOWARD F. PECKWORTH, *Chairman*

PHILIP C. RUTLEDGE, *Vice-Chairman*

E. LELAND DURKEE
TILTON E. SHELBURNE
R. ROBINSON ROWE
LOUIS E. RYDELL